UPDATED
7th EDITION

NEW SYLLABUS MATHEMATICS

WORKBOOK FULL SOLUTIONS

A Comprehensive Mathematics Programme for Grade 6

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Consultant • Dr Yeap Ban Har **Authors** • Dr Joseph Yeo • Teh Keng Seng • Loh Cheng Yee • Ivy Chow • Neo Chai Meng • Jacinth Liew

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ANSWERS

Chapter 1 Primes, Highest Common Factor and Lowest Common Multiple

Basic

1. **(a)** 101 is an odd number, so it is not divisible by 2. Since the sum of the digits 1 + 0 + 1 = 2 is not divisible by 3 (divisibility test for 3), then 101 is not divisible by 3. The last digit of 101 is neither 0 nor 5, so 101 is not divisible by 5.

A calculator may be used to test whether 101 is divisible by prime numbers more than 5. Since 101 is not divisible by any prime numbers less than 101, 101 is a prime number.

(b) 357 is an odd number, so it is not divisible by 2. Since the sum of the digits 3 + 5 + 7 = 15 which is divisible by 3, therefore 357 is divisible by 3 (divisibility test for 3).

$\therefore$ 357 is a composite number.

(c) 411 is an odd number, so it is not divisible by 2. Since the sum of the digits 4 + 1 + 1 = 6 which is divisible by 3, therefore 411 is divisible by 3 (divisibility test for 3).

$\therefore$ 411 is a composite number.

(d) 1223 is an odd number, so it is not divisible by 2. Since the sum of the digits 1 + 2 + 2 + 3 = 8 which is not divisible by 3, then 1223 is not divisible by 3. The last digit of 1223 is neither 0 nor 5, so 1223 is not divisible by 5.

A calculator may be used to test whether 1223 is divisible by prime numbers more than 5. Since 1223 is not divisible by any prime numbers less than 1223, 1223 is a prime number.

(e) 1555 is an odd number, so it is not divisible by 2. Since the sum of the digits 1 + 5 + 5 + 5 = 16 which is not divisible by 3, so 1555 is not divisible by 3. The last digit of 1555 is 5, so 1555 is divisible by 5.

$\therefore$ 1555 is a composite number.

(f) 3127 is an odd number, so it is not divisible by 2. Since the sum of the digits 3 + 1 + 2 + 7 = 13, then 3127 is not divisible by 3. A calculator may be used to test whether 3127 is divisible by prime numbers more than 3 and 3127 is divisible by 53, which is a prime number.

$\therefore$ 3127 is a composite number.

2. The prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Sum of prime numbers less than 30

= 2 + 3 + 5 +7 + 11 + 13 + 17 + 19 + 23 + 29

= 129

3. The two prime numbers between 20 and 30 are 23 and 29.

Difference of the two prime numbers = 29 – 23 = 6.

4. **(a)** Divide 315 by the smallest prime factor and continue the process until we obtain 1.

3	315
3	105
5	35
7	7
	1

$315 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7$

(b)

2	8008
2	4004
2	2002
7	1001
11	143
13	13
	1

$8008 = 2 \times 2 \times 2 \times 7 \times 11 \times 13$

$= 2^3 \times 7 \times 11 \times 13$

(c)

2	6120
2	3060
2	1530
3	765
3	255
5	85
17	17
	1

$61200 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 17$

$= 2^3 \times 3^2 \times 5 \times 17$

(d)

5	5875
5	1175
5	235
47	47
	1

$5875 = 5 \times 5 \times 5 \times 47$

$= 5^3 \times 47$

(e)

2	1780
2	890
5	445
89	89
	1

$1780 = 2 \times 2 \times 5 \times 89$

$= 2^2 \times 5 \times 89$

5. (a) $16 = 2 \times 2 \times 2 \times 2$

$24 = 2 \times 2 \times 2 \times 3$

2 2 2

HCF of 16 and 24 $= 2 \times 2 \times 2 = 8$

(b) $45 = 3 \times 3 \times 5$

$63 = 3 \times 3 \times 7$

3 3

HCF of 45 and 63 $= 3 \times 3 = 9$

(c) $56 = 2 \times 2 \times 2 \times 7$

$70 = 2 \times 5 \times 7$

2 7

HCF of 56 and 70 $= 2 \times 7 = 14$

(d) $90 = 2 \times 3 \times 3 \times 5$

$126 = 2 \times 3 \times 3 \times 7$

2 3 3

HCF of 90 and 126 $= 2 \times 3 \times 3 = 18$

(e) $108 = 2 \times 2 \times 3 \times 3 \times 3$

$196 = 2 \times 2 \times 7 \times 7$

2 2

HCF of 108 and 196 $= 2 \times 2 = 4$

(f) $108 = 2 \times 2 \times 3 \times 3 \times 5$

$158 = 2 \times 79$

2

HCF of 108 and 158 = 2

(g) $42 = 2 \times 3 \times 7$

$66 = 2 \times 3 \times 11$

$78 = 2 \times 3 \times 13$

2 3

HCF of 42, 66 and 78 $= 2 \times 3 = 6$

(h) $132 = 2 \times 2 \times 3 \times 11$

$156 = 2 \times 2 \times 3 \times 13$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

2 2 3

HCF of 132, 156 and 180 $= 2 \times 2 \times 3 = 12$

(i) $84 = 2 \times 2 \times 3 \times 7$

$98 = 2 \times 7 \times 7$

$112 = 2 \times 2 \times 2 \times 2 \times 7$

2 7

HCF of 84, 98 and 112 $= 2 \times 7 = 14$

(j) $195 = 3 \times 5 \times 13$

$270 = 2 \times 3 \times 3 \times 3 \times 5$

$345 = 3 \times 5 \times 23$

3 5

HCF of 195, 270 and 345 $= 3 \times 5 = 15$

(k) $147 = 3 \times 7 \times 7$

$231 = 3 \times 7 \times 11$

$273 = 3 \times 7 \times 13$

3 7

HCF of 147, 231 and 273 $= 3 \times 7 = 21$

(l) $225 = 3 \times 3 \times 5 \times 5$
$495 = 3 \times 3 \times 5 \times 11$
$810 = 2 \times 3 \times 3 \times 3 \times 5$
3 3 5

HCF of 225, 495 and 810 = $3 \times 3 \times 5 = 45$

6. (a) $48 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$72 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
2 2 2 2 3 3

LCM of 48 and 72 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

(b) $75 = 3 \times 5 \times 5$
$105 = 3 \times 5 \times 7$
3 5 5 7

LCM of 75 and 105 = $3 \times 5 \times 5 \times 7 = 525$

(c) $243 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$405 = 3 \times 3 \times 3 \times 3 \times 5$
3 3 3 3 3 5

LCM of 243 and 405 = $3 \times 3 \times 3 \times 3 \times 3 \times 5$
$= 1215$

(d) $261 = 3 \times 3 \times 29$
$435 = 3 \times 5 \times 29$
3 3 5 29

LCM of 261 and 435 = $3 \times 3 \times 5 \times 29 = 1305$

(e) $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$306 = 2 \times 3 \times 3 \times 17$
2 2 2 2 3 3 17

LCM of 144 and 306 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 17$
$= 2448$

(f) $264 = 2 \times 2 \times 2 \times 3 \times 11$
$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$
2 2 2 3 3 7 11

LCM of 264 and 504 = $2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 11$
$= 5544$

(g) $176 = 2 \times 2 \times 2 \times 2 \times 11$
$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5$
2 2 2 2 11 2 5

LCM of 176 and 160 = $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 11$
$= 3520$

(h) $56 = 2 \times 2 \times 2 \times 7$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$104 = 2 \times 2 \times 2 \times 13$
2 2 2 3 3 7 13

LCM of 56, 72 and 104
$= 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 13 = 6552$

(i) $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$
$972 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$
2 2 3 3 3 3 3 7

LCM of 324, 756 and 972
$= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7$
$= 6804$

(j) $450 = 2 \times 3 \times 3 \times 5 \times 5$
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$180 = 2 \times 2 \times 3 \times 3 \times 5$
2 2 2 2 3 3 5 5

LCM of 450, 720 and 180
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= 3600$

7. (a) $84 = 2 \times 2 \times 3 \times 7$
$189 = 3 \times 3 \times 7$
3 7

HCF of 84 and 189 = $3 \times 7 = 21$

$84 = 2 \times 2 \times 3 \times 7$
$189 = 3 \times 3 \times 7$
2 2 3 3 3 7

LCM of 84 and 189 = $2 \times 2 \times 3 \times 3 \times 3 \times 7$
$= 756$

(b) $315 = 3 \times 3 \times 5 \times 7$
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
3 3 5

HCF of 315 and 720 = $3 \times 3 \times 5 = 45$

$315 = 3 \times 3 \times 5 \times 7$
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
2 2 2 2 3 3 5 7

LCM of 315 and 720
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 5040$

(c) $392 = 2 \times 2 \times 2 \times 7 \times 7$
$616 = 2 \times 2 \times 2 \times 7 \times 11$
2 2 2 7

HCF of 392 and 616 $= 2 \times 2 \times 2 \times 7 = 56$

$392 = 2 \times 2 \times 2 \times 7 \times 7$
$616 = 2 \times 2 \times 2 \times 7 \times 11$
2 2 2 7 7 11

LCM of 392 and 616 $= 2 \times 2 \times 2 \times 7 \times 7 \times 11$
$= 4312$

(d) $100 = 2 \times 2 \times 5 \times 5$
$164 = 2 \times 2 \times 41$
2 2

HCF of 100 and 164 $= 2 \times 2$
$= 4$

$100 = 2 \times 2 \times 5 \times 5$
$164 = 2 \times 2 \times 41$
2 2 5 5 41

LCM of 100 and 164
$= 2 \times 2 \times 5 \times 4 \times 41$
$= 4100$

(e) $140 = 2 \times 2 \times 5 \times 7$
$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$
$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$
2 2 7

HCF of 140, 224 and 560 $= 2 \times 2 \times 7 = 28$

$140 = 2 \times 2 \times 5 \times 7$
$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$
$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$
2 2 2 2 2 5 7

LCM of 140, 224 and 560
$= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 = 1120$

(f) $315 = 3 \times 3 \times 5 \times 7$
$525 = 3 \times 5 \times 5 \times 7$
$140 = 2 \times 2 \times 5 \times 7$
5 7

HCF of 315, 525 and 140 $= 5 \times 7 = 35$

$315 = 3 \times 3 \times 5 \times 7$
$525 = 3 \times 5 \times 5 \times 7$
$140 = 2 \times 2 \times 5 \times 7$
2 2 2 3 3 5 5 7

LCM of 315, 525 and 140
$= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$
$= 6300$

(g) $252 = 2 \times 2 \times 3 \times 3 \times 7$
$378 = 2 \times 3 \times 3 \times 3 \times 7$
$567 = 3 \times 3 \times 3 \times 3 \times 7$
3 3 7

HCF of 252, 378 and 567 $= 3 \times 3 \times 7 = 63$

$252 = 2 \times 2 \times 3 \times 3 \times 7$
$378 = 2 \times 3 \times 3 \times 3 \times 7$
$567 = 3 \times 3 \times 3 \times 3 \times 7$
2 2 3 3 3 3 7

LCM of 252, 378 and 567
$= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 = 2268$

(h) $330 = 2 \times 3 \times 5 \times 11$
$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$
$114 = 2 \times 3 \times 19$
2 3

HCF of 330, 792 and 114 $= 2 \times 3 = 6$

$330 = 2 \times 3 \times 5 \times 11$
$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$
$114 = 2 \times 3 \times 19$
2 2 2 3 3 5 11 19

LCM of 330, 792 and 114
$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11 \times 19$
$= 75240$

8. (a) $2^2 \times 3^2 \times 11$
$2^4 \times 3 \times 7$
2^2 3

HCF $= 2^2 \times 3$

$2^2 \times 3^2 \times 11$
$2^4 \times 3 \times 7$
2^4 3^2 7 11

LCM $= 2^4 \times 3^2 \times 7 \times 11$

(b) $3^4 \times 5^2 \times 7 \times 7^3 \times 11$

$3^3 \quad 7$

HCF = $3^3 \times 7$

$3^4 \quad 5^2 \quad 7^3 \quad 11$

LCM = $3^4 \times 5^2 \times 7^3 \times 11$

(c) $7290 = 2 \times 3^6 \times 5 \times 2^3 \times 3^3 \times 5^6$

$2 \quad 3^3 \quad 5$

HCF = $2 \times 3^3 \times 5$

$7290 = 2 \times 3^6 \times 5 \times 2^3 \times 3^3 \times 5^6$

$2^3 \quad 3^6 \quad 5^6$

LCM = $2^3 \times 3^6 \times 5^6$

(d) $1606 = 2 \times 11 \times 73 \times 3^5 \times 7^6 \times 11$

11

HCF = 11

$1606 = 2 \times 11 \times 73 \times 3^5 \times 7^6 \times 11$

$2 \quad 3^5 \quad 7^6 \quad 11 \quad 73$

LCM = $2 \times 3^5 \times 7^6 \times 11 \times 73$

Intermediate

9. $720 = 2^4 \times 3^2 \times 5$

$1575 = 3^2 \times 5^2 \times 7$

(i) Largest prime factor of 720 and 1575 = 5

(ii) LCM of 720 and 1575 = $2^4 \times 3^2 \times 5^2 \times 7$

= 25 200

10. $374 = 2 \times 11 \times 17$

$34 = 2 \times 17$

So the smallest number that gives LCM of 374 is 11.

Thus $m = 11$.

11. Divide 1764 by the smallest prime number until we get 1.

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$1764 = 2^2 \times 3^2 \times 7^2 = (2 \times 3 \times 7)^2$

$\sqrt{1764} = 2 \times 3 \times 7 = 42$

12. $84 = 2 \times 2 \times 3 \times 7$

$126 = 2 \times 3 \times 3 \times 7$

(i) To find the length of each square is to find the largest whole number which is a factor of both 84 and 126.

$84 = 2 \times 2 \times 3 \times 7$

$126 = 2 \times 3 \times 3 \times 7$

$2 \quad 3 \quad 7$

HCF of 84 and 126 = $2 \times 3 \times 7 = 42$

Thus the length of each square is 42 cm.

(ii) Area of the rectangular sheet = 84×126

= 10 584 cm^2

Area of each square = $42 \times 42 = 1764$ cm^2

Number of squares that she can cut

= $10\,584 \div 1764 = 6$

13. $48 = 2 \times 2 \times 2 \times 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$2 \quad 2 \quad 2 \quad 3$

(i) Greatest number of discussion topics

= HCF of 48, 72 and 96

$= 2 \times 2 \times 2 \times 3$

= 24

(ii) Number of participants from China in each discussion group

$= 96 \div 24$

= 4

14. $8 = 2 \times 2 \times 2$

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

$2 \quad 2 \quad 2 \quad 3 \quad 5$

∴ LCM of 8, 10 and 12 = $2 \times 2 \times 2 \times 3 \times 5 = 120$

∴ The three canteens will serve noodle soup again after 120 days.

15. $160 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times 5$
$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
2 2 2 2

(i) Largest possible length of each piece of ribbon
$= 2 \times 2 \times 2 \times 2$
$= 16$ cm

(ii) Total number of ribbons
$= (160 \div 16) + (192 \div 16) + (240 \div 16)$
$= 37$

16. To find the time when they next meet again is to find the LCM of 126, 154 and 198 seconds.

$126 = \boxed{2} \times \boxed{3} \times \boxed{3} \times \boxed{7}$
$154 = 2 \times 7 \times 11$
$198 = \boxed{2} \times \boxed{3} \times \boxed{3} \times 11$
2 3 3 7 11

LCM of 126, 154 and 198 $= 2 \times 3 \times 3 \times 7 \times 11$
$= 1386$

Time when they next meet again
= 4 pm + 23 min 6 s
= 4.23 pm

17. (i) To find the greatest number of hampers that can be packed is to find the HCF of the boxes of chocolates, the bottles of wine and the tins of biscuits.

$420 = \boxed{2} \times 2 \times \boxed{3} \times 5 \times 7$
$630 = 2 \times 3 \times 3 \times 5 \times 7$
$1260 = \boxed{2} \times 2 \times \boxed{3} \times 3 \times 5 \times 7$
2 3 5 7

HCF of 420, 630 and 1260 $= 2 \times 3 \times 5 \times 7 = 210$

(ii) Number of boxes of chocolate $= 1260 \div 210$
$= 6$

Number of bottles of wine $= 420 \div 210 = 2$
Number of tins of biscuits $= 630 \div 210 = 3$

Advanced

18. (i) $15 = 3 \times 5$
$20 = 2 \times 2 \times 5$
$27 = 3 \times 3 \times 3$
LCM of 15, 20 and 27 $= 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 540$

The next event will happen 540 seconds or 9 minutes later, i.e. at 12.09 am.

(ii) Since it happens after every 9 minutes and there are 60 minutes between midnight and 1 am, it will happen for another 6 times.

19. $24 = 2 \times 2 \times 2 \times 3$
$42 = 2 \times 3 \times 7$
$60 = 2 \times 2 \times 3 \times 5$
LCM of 24, 42 and 60 $= 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$
Shortest possible length = 840 cm

20. $36 = 2 \times 2 \times 3 \times 3$
$56 = 2 \times 2 \times 2 \times 7$
$1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$
Smallest value of n
$= 3 \times 3 \times 3$
$= 27$

21. $A = 2^2 \times 3^4 \times 5^2 \times 7^4 \times 13^3$
$B = 2^4 \times 3^6 \times 5^2 \times 7^5 \times 11^{16}$
$C = 3^7 \times 5^2 \times 7 \times 17^2$

(a) HCF of A, B and $C = 3^4 \times 5^2 \times 7$

(b) LCM of A, B and C
$= 2^4 \times 3^7 \times 5^2 \times 7^5 \times 11^{16} \times 13^3 \times 17^2$

22. Consider multiples of 4 and they are 8, 12, 16 and 20. We can find the corresponding numbers which give HCF = 4 and LCM = 120.

Case 1
$4 = 2 \times 2$
LCM $= 2 \times 2 \times 30 = 120$. Thus the next number is $2 \times 2 \times 30 = 120$.
The first set of numbers is 4 and 120.

Case 2
$8 = 2 \times 2 \times 2$
LCM $= 2 \times 2 \times 2 \times 15 = 120$. Thus the next number is $2 \times 2 \times 15 = 60$.
The second set of numbers is 8 and 60.

Case 3
$12 = 2 \times 2 \times 3$
LCM $= 2 \times 2 \times 3 \times 10 = 120$. Thus the next number is $2 \times 2 \times 10 = 40$.
The third set of numbers is 12 and 40.

Case 4

$20 = 2 \times 2 \times 5$

LCM = $2 \times 2 \times 5 \times 6 = 120$. Thus the next number is $2 \times 2 \times 6 = 24$.

The last set of numbers is 20 and 24.

23. By observation, $19 \times 11 = 209$ where $19 + 11 = 30$ but 209 does not contain all prime numbers.

So, we can try $19 \times 2 \times 3 \times 5$. But $19 + 2 + 3 + 5 \neq 30$ and $19 \times 2 \times 3 \times 5 = 570$ and 0 is not a prime number.

Therefore we can try $19 \times 2 \times 2 \times 7$.

$19 + 2 + 2 + 7 = 30$

and $19 \times 2 \times 2 \times 7 = 532$ and 5, 3 and 2 are prime numbers.

So, the 3-digit number that satisfies all the conditions is 532.

New Trend

24. (a) $504 = 2^3 \times 3^2 \times 7$

(b) HCF: 2×3

LCM: $2^3 \times 3^2 \times 7$

First number = $2 \times 3 \times 7 = 42$

Second number = $2^3 \times 3^2 = 72$

25. (a)

2	3234
3	1617
7	537
7	77
11	11
	1

$3234 = 2 \times 3 \times 7 \times 7 \times 11 = 2 \times 3 \times 7^2 \times 11$

(b) $4 = 2 \times 2$

$30 = 2 \times 3 \times 5$

LCM of 4 and 30 = $2 \times 2 \times 3 \times 5$

$= 60$

Factors of 60 = 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Chapter 2 Integers and Order of Operations

Basic

1. **(a)** If –15 represents 15 m below sea level, then +20 represents 20 m above sea level.

(b) If –10 represents the distance of 10 km of a car travelling south, then +10 represents the distance of 10 km of a car travelling north.

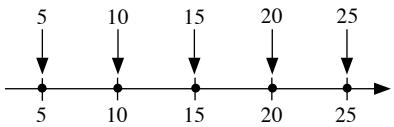
(c) If +100 represents a profit of PKR 100 on the sale of a mobile phone, then –91 represents a loss of PKR 91 on the sale.

(d) If +90° represents a clockwise rotation of 90°, then –90° represents rotating 90° anticlockwise.

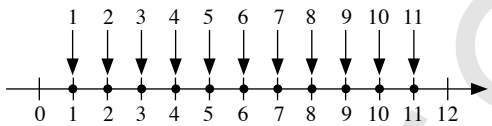
(e) If –5 represents 5 flights down the stairs, then 14 flights up the stairs is represented by +14.

(f) If +600 represents a deposit of PKR 600 in the bank, then a withdrawal of PKR 60 is represented by –60.

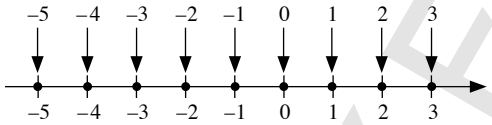
2. **(a)**



(b)



(c)



3. **(a)** $6 \boxed{>} -6$

(b) $0 \boxed{>} -\frac{4}{5}$

(c) $-12 \boxed{>} -16$

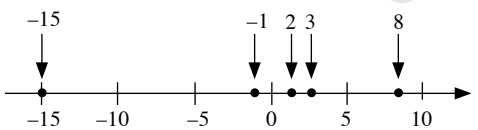
(d) $-\frac{12}{2} = -6$

Since $-6 < -5$,

Therefore, $-\frac{12}{2} \boxed{<} -5$

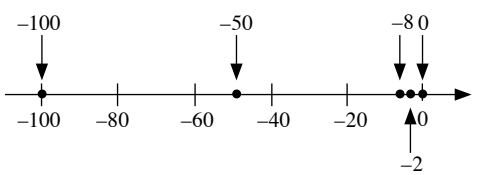
(e) $-5.6 \boxed{<} -3.4$

4. **(a)**



$\therefore$ –15, –1, 2, 3, 8

(b)



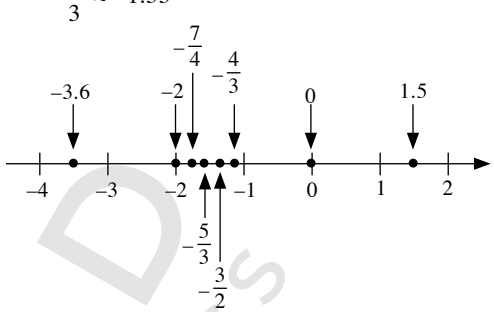
$\therefore$ –100, –50, –8, –2, 0

(c) $-\frac{7}{4} = -1.75$

$-\frac{5}{3} \approx -1.67$

$-\frac{3}{2} = -1.5$

$-\frac{4}{3} \approx -1.33$



$\therefore -3.6, -2, -\frac{7}{4}, -\frac{5}{3}, -\frac{3}{2}, -\frac{4}{3}, 0, 1.5$

5. **(a)** $-5 + 13 = 13 - 5$
$= 8$

(b) $-25 + 12 = 12 - 25$
$= -13$

(c) $5 + (-4) = 5 - 4$
$= 1$

(d) $19 + (-26) = 19 - 26$
$= -7$

(e) $-2 + (-2) = -2 - 2$
$= -4$

(f) $-5 + (-3) = -5 - 3$
$= -8$

(g) $-11 + (-10) = -11 - 10$
$= -21$

(h) $-25 + (-65) = -25 - 65$
$= -90$

6. **(a)** $14 - 18 = -4$

(b) $-5 - 3 = -8$

(c) $-12 - 13 = -25$

(d) $-(-13) = 13$

(e) $6 - (-11) = 6 + 11$
$= 17$

(f) $-8 - (-11) = -8 + 11$
$= 11 - 8$
$= 3$

(g) $(-17) - (-35) = -17 + 35$
$= 35 - 17$
$= 18$

(h) $(-25) - (-10) = -25 + 10$
$= 10 - 25$
$= -15$

7. **(a)** $5 \times (-4) = 5 \times (-1 \times 4)$
$= 5 \times (-1) \times 4$
$= (-1) \times 20$
$= -20$

(b) $-3 \times 8 = (-1 \times 3) \times 8$
$= (-1) \times 3 \times 8$
$= (-1) \times 24$
$= -24$

(c) $(-4) \times (-12) = (-1 \times 4) \times (-12)$
$= (-1 \times 4) \times (-1 \times 12)$
$= (-1) \times 4 \times (-1) \times 12$
$= (-1) \times (-1) \times 4 \times 12$
$= 1 \times 48$
$= 48$

(d) $-5(-16) = (-1 \times 5) \times (-16)$
$= (-1 \times 5) \times (-1 \times 16)$
$= (-1) \times 5 \times (-1) \times 16$
$= (-1) \times (-1) \times 5 \times 16$
$= 1 \times 80$
$= 80$

(e) $-10(-20) = (-1 \times 10) \times (-20)$
$= (-1 \times 10) \times (-1 \times 20)$
$= (-1) \times 10 \times (-1) \times 20$
$= (-1) \times (-1) \times 10 \times 20$
$= 1 \times 200$
$= 200$

(f) $0 \times (-18) = 0 \times (-1) \times 18$
$= (-1) \times 0 \times 18$
$= (-1) \times 0$
$= 0$

(g) $56 \div (-7) = \frac{56}{-7}$
$= 56 \times \frac{1}{-7}$
$= 56 \times \left(-\frac{1}{7}\right)$
$= -8$

(h) $0 \div (-12) = \frac{0}{-12}$
$= 0 \times \frac{1}{-12}$
$= 0$

(i) $-100 \div (-4) = \frac{-100}{-4}$
$= -100 \times \frac{1}{-4}$
$= -100 \times \left(-\frac{1}{4}\right)$
$= 25$

(j) $(-75) \div (-25) = \frac{-75}{-25}$
$= -75 \times \frac{1}{-25}$
$= -75 \times \left(-\frac{1}{25}\right)$
$= 3$

(k) $\frac{70}{-14} = 70 \times \frac{1}{-14}$
$= 70 \times \left(-\frac{1}{14}\right)$
$= -5$

(l) $\frac{-90}{-15} = -90 \times \frac{1}{-15}$
$= -90 \times \left(-\frac{1}{15}\right)$
$= 6$

8. **(a)** $(-2) \times (-3) \times (-4) \times (-5) = 6 \times (-4) \times (-5)$
$= -(6 \times 4) \times (-5)$
$= (-24) \times (-5)$
$= 120$

(b) $(-8) \times (-3) \times 5 \times (-6) = 24 \times 5 \times (-6)$
$= 120 \times (-6)$
$= -(120 \times 6)$
$= -720$

(c) $(-2) \times 5 \times (-9) \times (-7) = -(2 \times 5) \times (-9) \times (-7)$
$= -10 \times (-9) \times (-7)$
$= 90 \times (-7)$
$= -630$

(d) $4 \times (-4) \times (-5) \times (-16)$
$= -(4 \times 4) \times (-5) \times (-16)$
$= -16 \times (-5) \times (-16)$
$= 80 \times (-16)$
$= -1280$

(e) $5 \times 6 \times (-1) \times (-12) = 30 \times (-1) \times (-12)$
$= (-30 \times 1) \times (-12)$
$= -30 \times (-12)$
$= 360$

(f) $(-1) \times (-8) \times 3 \times 5 = 8 \times 3 \times 5$
$= 24 \times 5$
$= 120$

(g) $140 \div (-7) \div 4 = \left(\frac{140}{-7}\right) \div 4$
$= \left(140 \times \frac{-1}{7}\right) \div 4$
$= (-20) \div 4$
$= \frac{-20}{4}$
$= -20 \times \left(\frac{1}{4}\right)$
$= -5$

(h) $(-264) \div 11 \div 8 = \left(\frac{-264}{11}\right) \div 8$

$= \left(-264 \times \frac{1}{11}\right) \div 8$

$= (-24) \div 8$

$= -3$

(i) $(-390) \div (-13) \div (-5) = \left(\frac{-390}{-13}\right) \div (-5)$

$= \left(-390 \times \frac{1}{-13}\right) \div (-5)$

$= (30) \div (-5)$

$= -(30 \div 5)$

$= -6$

(j) $(-9) \times (-4) \div (-12) = (36) \div (-12)$

$= \left(\frac{36}{-12}\right)$

$= (36) \times \left(-\frac{1}{12}\right)$

$= -3$

(k) $(-56) \div (-8) \times 15 = \left(\frac{-56}{-8}\right) \times 15$

$= \left(-56 \times \frac{1}{-8}\right) \times 15$

$= 7 \times 15$

$= 105$

(l) $\sqrt{-288 \div (-2)} \times (-3)^2$

$= \left(\sqrt{\frac{-288}{-2}}\right) \times (-3)^2$

$= \left(\sqrt{-288 \times \left(\frac{1}{-2}\right)}\right) \times (-3)^2$

$= \left(\sqrt{144}\right) \times (-3)^2$

$= 12 \times (-3)^2$

$= 12 \times 9$

$= 108$

9. **(a)** $(-2) \times (-3) \times (-4) \times (-5) = 120$

(b) $(-8) \times (-3) \times 5 \times (-6) = -720$

(c) $(-2) \times 5 \times (-9) \times (-7) = -630$

(d) $4 \times (-4) \times (-5) \times (-16) = -1280$

(e) $5 \times 6 \times (-1) \times (-12) = 360$

(f) $(-1) \times (-8) \times 3 \times 5 = 120$

(g) $140 \div (-7) \div 4 = -5$

(h) $(-264) \div 11 \div 8 = -3$

(i) $(-390) \div (-13) \div (-5) = -6$

(j) $(-9) \times (-4) \div (-12) = -3$

(k) $(-56) \div (-8) \times 15 = 105$

(l) $\sqrt{-288 \div (-2)} \times (-3)^2 = 108$

10. **(a)** $[(-3) + (-4)] \div 7 = [(-3) - 4] \div 7$

$= (-7) \div 7$

$= \frac{-7}{7}$

$= -7 \times \frac{1}{7}$

$= -1$

(b) $(-56) \div [7 + (-14)] = (-56) \div [7 - 14]$

$= (-56) \div (-7)$

$= \frac{-56}{-7}$

$= -56 \times \left(-\frac{1}{7}\right)$

$= 8$

(c) $(-72) \div [-14 - (-23)] = (-72) \div (-14 + 23)$

$= (-72) \div (9)$

$= \frac{-72}{9}$

$= -72 \times \frac{1}{9}$

$= -8$

(d) $32 + (-16) \div (-2)^2 = 32 + (-16) \div 4$

$= 32 + \left(\frac{-16}{4}\right)$

$= 32 + \left(-16 \times \frac{1}{4}\right)$

$= 32 + (-4)$

$= 32 - 4$

$= 28$

(e) $(47 + 19 - 36) \div (-5) = (66 - 36) \div (-5)$

$= 30 \div (-5)$

$= \frac{30}{-5}$

$= 30 \times -\frac{1}{5}$

$= -6$

11. **(a)** $[(-3) + (-4)] \div 7 = -1$

(b) $(-56) \div [7 + (-14)] = 8$

(c) $(-72) \div [-14 - (-23)] = -8$

(d) $32 + (-16) \div (-2)^2 = 28$

(e) $(47 + 19 - 36) \div (-5) = -6$

12. (a) $2\frac{5}{9} - 3\frac{1}{4} = \frac{23}{9} - \frac{13}{4}$

$= \frac{23 \times 4}{9 \times 4} - \frac{13 \times 9}{4 \times 9}$

$= \frac{92}{36} - \frac{117}{36}$

$= \frac{92 - 117}{36}$

$= -\frac{25}{36}$

(b) $2\frac{1}{4} + \left(-1\frac{3}{5}\right) = 2\frac{1}{4} - 1\frac{3}{5}$

$= \frac{9}{4} - \frac{8}{5}$

$= \frac{9 \times 5}{4 \times 5} - \frac{8 \times 4}{5 \times 4}$

$= \frac{45 - 32}{20}$

$= \frac{13}{20}$

(c) $9\frac{1}{4} + \left(-7\frac{3}{5}\right) = 9\frac{1}{4} - 7\frac{3}{5}$

$= \frac{37}{4} - \frac{38}{5}$

$= \frac{37 \times 5}{4 \times 5} - \frac{38 \times 4}{5 \times 4}$

$= \frac{185 - 152}{20}$

$= \frac{33}{20}$

$= 1\frac{13}{20}$

(d) $\frac{2}{5} - \left(-\frac{1}{6}\right) = \frac{2}{5} + \frac{1}{6}$

$= \frac{2 \times 6}{5 \times 6} + \frac{1 \times 5}{6 \times 5}$

$= \frac{12 + 5}{30}$

$= \frac{17}{30}$

(e) $-1\frac{1}{5} + \left(-1\frac{1}{3}\right) = -\frac{6}{5} - 1\frac{1}{3}$

$= -\frac{6}{5} - \frac{4}{3}$

$= \frac{-6 \times 3}{5 \times 3} - \frac{4 \times 5}{3 \times 5}$

$= \frac{-18 - 20}{15}$

$= \frac{-38}{15}$

$= -2\frac{8}{15}$

(f) $-2\frac{1}{3} - \left(-1\frac{1}{2}\right) = -2\frac{1}{3} + 1\frac{1}{2}$

$= -\frac{7}{3} + \frac{3}{2}$

$= \frac{-7 \times 2}{3 \times 2} + \frac{3 \times 3}{2 \times 3}$

$= \frac{-14 + 9}{6}$

$= -\frac{5}{6}$

(g) $-4\frac{2}{9} - \left(-1\frac{1}{6}\right) = -4\frac{2}{9} + 1\frac{1}{6}$

$= -\frac{38}{9} + \frac{7}{6}$

$= \frac{-38 \times 6}{9 \times 6} + \frac{7 \times 9}{6 \times 9}$

$= \frac{-228 + 63}{54}$

$= -\frac{165}{54}$

$= -\frac{55}{18}$

(h) $-\left(-\frac{7}{8}\right) - 1\frac{3}{4} = \frac{7}{8} - 1\frac{3}{4}$

$= \frac{7}{8} - \frac{7}{4}$

$= \frac{7 \times 4}{8 \times 4} - \frac{7 \times 8}{4 \times 8}$

$= \frac{28 - 56}{32}$

$= -\frac{28}{32}$

$= -\frac{7}{8}$

13. (a) $2\frac{5}{9} - 3\frac{1}{4} = -\frac{25}{36}$

(b) $2\frac{1}{4} + \left(-1\frac{3}{5}\right) = \frac{13}{20}$

(c) $9\frac{1}{4} + \left(-7\frac{3}{5}\right) = 1\frac{13}{20}$

(d) $\frac{2}{5} - \left(-\frac{1}{6}\right) = \frac{17}{30}$

(e) $-1\frac{1}{5} + \left(-1\frac{1}{3}\right) = -2\frac{8}{15}$

(f) $-2\frac{1}{3} - \left(-1\frac{1}{2}\right) = -\frac{5}{6}$

(g) $-4\frac{2}{9} - \left(-1\frac{1}{6}\right) = -\frac{55}{18}$

(h) $-\left(-\frac{7}{8}\right) - 1\frac{3}{4} = -\frac{7}{8}$

14. **(a)** $5 \times \left(-2\frac{2}{5}\right) = {}^{1}\cancel{5} \times \left(\frac{-12}{\cancel{5}_{1}}\right)$

$= -12$

(b) $\left(-\frac{4}{5}\right) \div (-16) = \left(-\frac{{}^{1}\cancel{4}}{5}\right) \times \left(\frac{-1}{\cancel{16}_{4}}\right)$

$= \frac{1}{20}$

(c) $16\frac{3}{10} \times \left(-\frac{5}{8}\right) = \left(\frac{163}{{}_{2}\cancel{10}}\right) \times \left(-\frac{\cancel{5}^{1}}{8}\right)$

$= -\frac{163}{16}$

$= -10\frac{3}{16}$

(d) $-\frac{4}{9} \times \frac{3}{14} = -\frac{{}^{2}\cancel{4}}{{}_{3}\cancel{9}} \times \frac{\cancel{3}^{1}}{\cancel{14}_{7}}$

$= -\frac{2}{21}$

(e) $\left(-3\frac{1}{2}\right) \times 2\frac{3}{5} = -\frac{7}{2} \times \frac{13}{5}$

$= -\frac{91}{10}$

$= -9\frac{1}{10}$

(f) $\left(-7\frac{1}{3}\right) \div 1\frac{5}{6} = -\frac{22}{3} \div \frac{11}{6}$

$= -\frac{{}^{2}\cancel{22}}{{}_{1}\cancel{3}} \times \frac{\cancel{6}^{2}}{\cancel{11}_{1}}$

$= -4$

(g) $-\frac{{}^{1}\cancel{7}}{{}_{2}\cancel{18}} \times \left(-\frac{\cancel{9}^{1}}{\cancel{14}_{2}}\right) = -\frac{1}{2} \times \left(-\frac{1}{2}\right)$

$= -\left(-\frac{1}{4}\right)$

$= \frac{1}{4}$

(h) $\left(-\frac{5}{6}\right) \div \left(-1\frac{3}{4}\right) = -\frac{5}{6} \div \left(-\frac{7}{4}\right)$

$= -\frac{5}{{}_{3}\cancel{6}} \times \left(-\frac{\cancel{4}^{2}}{7}\right)$

$= \frac{10}{21}$

15. **(a)** $5 \times \left(-2\frac{2}{5}\right) = -12$

(b) $\left(-\frac{4}{5}\right) \div (-16) = \frac{1}{20}$

(c) $16\frac{3}{10} \times \left(-\frac{5}{8}\right) = -10\frac{3}{16}$

(d) $-\frac{4}{9} \times \frac{3}{14} = -\frac{2}{21}$

(e) $\left(-3\frac{1}{2}\right) \times 2\frac{3}{5} = -9\frac{1}{10}$

(f) $\left(-7\frac{1}{3}\right) \div 1\frac{5}{6} = -4$

(g) $-\frac{{}^{1}\cancel{7}}{{}_{2}\cancel{18}} \times \left(-\frac{\cancel{9}^{1}}{\cancel{14}_{2}}\right) = \frac{1}{4}$

(h) $\left(-\frac{5}{6}\right) \div \left(-1\frac{3}{4}\right) = \frac{10}{21}$

16. **(a)**

$$\begin{array}{r} 14.8 \\ \times\quad 6.2 \\ \hline 296 \\ +888 \\ \hline 91.76 \end{array}$$

$\therefore 14.8 \times 6.2 = 91.76$

(b)

$$\begin{array}{r} 144.735 \\ \times\quad 0.15 \\ \hline 723675 \\ +1447350 \\ \hline 21.710025 \end{array}$$

$\therefore 144.735 \times 0.15 = 21.710\,25$

(c)

$$\begin{array}{r} 0.35 \\ \times\quad 0.096 \\ \hline 210 \\ +\ 315 \\ \hline 0.03360 \end{array}$$

$\therefore 0.35 \times 0.096 = 0.0336$

(d)

$$\begin{array}{r} 1.84 \\ \times\quad 0.092 \\ \hline 368 \\ +1656 \\ \hline 0.16928 \end{array}$$

$\therefore 1.84 \times 0.092 = 0.169\,28$

(e) $1.45 \div 0.16$

$= \frac{1.45}{0.16}$

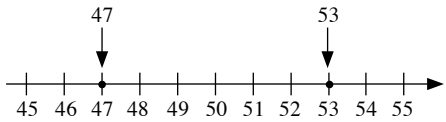
$= \frac{145}{16}$

$$\begin{array}{r} 9.0625 \\ 16\overline{)\ 145.0000} \\ -144 \\ \hline 100 \\ -\ 96 \\ \hline 40 \\ -32 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

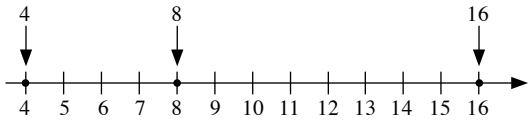
$\therefore 1.45 \div 0.16 = 9.0625$

Intermediate

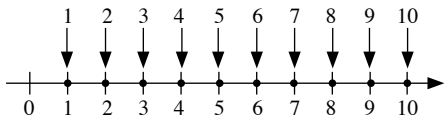
17. (a)



(b) Factors of 64 and 80 are 1, 2, 4, 5, 8 and 16. Composite numbers that are factors of both 64 and 80 are 4, 8, 16.



(c) Natural numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10……



18. (a) $4 + (-15) - 21$
$= 4 - 15 - 21$
$= -11 - 21$
$= -(11 + 21)$
$= -32$

(b) $-4 + (-12) + 10$
$= -4 - 12 + 10$
$= -16 + 10$
$= -(16 - 10)$
$= -6$

(c) $-5 + (-7) - (-13)$
$= -5 - 7 + 13$
$= -12 + 13$
$= -(12 - 13)$
$= -(-1)$
$= 1$

(d) $20 + (-9) - (-16)$
$= 20 - 9 - (-16)$
$= 20 - 9 + 16$
$= 11 + 16$
$= 27$

(e) $3 - (-7) - 4 + (-4)$
$= 3 + 7 - 4 - 4$
$= 10 - 4 - 4$
$= 6 - 4$
$= 2$

(f) $-27 - (-35) - 5 + (-9)$
$= -27 + 35 - 5 - 9$
$= -(27 - 35) - 5 - 9$
$= -(-8) - 5 - 9$
$= 8 - 5 - 9$
$= 3 - 9$
$= -6$

(g) $35 - (-5) + (-12) - (-8)$
$= 35 + 5 - 12 + 8$
$= 40 - 12 + 8$
$= 28 + 8$
$= 36$

(h) $23 + (-3) - (-7) + (-22)$
$= 23 - 3 + 7 - 22$
$= 20 + 7 - 22$
$= 27 - 22$
$= 5$

(i) $-14 - [-6 + (-15)]$
$= -14 - (-6 - 15)$
$= -14 - (-21)$
$= -14 + 21$
$= 7$

(j) $[-4 + (-14)] + [-8 - (-26)]$
$= (-4 - 14) + (-8 + 26)$
$= (-18) + (26 - 8)$
$= -18 + 18$
$= 0$

19. $[-2 + (-14) - 10] - [(-6)^2 + (-17) - (-9)]$
$= (-2 - 14 - 10) - [36 + (-17) - (-9)]$
$= (-26) - (36 - 17 + 9)$
$= -26 - 28$
$= -(26 + 28)$
$= -54$

20. (a) $\dfrac{(-2) \times (-5) + (-20)}{(-10)}$

$= \dfrac{10 + (-20)}{-10}$

$= \dfrac{10 - 20}{-10}$

$= \dfrac{-10}{-10}$

$= 1$

(b) $\dfrac{(-123) \times [19 + (-19)]}{38}$

$= \dfrac{(-123) \times (19 - 19)}{38}$

$= \dfrac{-123 \times 0}{38}$

$= \dfrac{0}{38}$

$= 0$

(c) $(-11) \times [-52 + (-17) - (-39)]$

$= (-11) \times (-52 - 17 + 39)$

$= (-11) \times (-69 + 39)$

$= (-11) \times (-30)$

$= -(-330)$

$= 330$

(d) $16 + (-21) \div 7 \times \{9 + [56 \div (-8)]\}$

$= 16 + (-21) \div 7 \times [9 + (-7)]$

$= 16 + (-21) \div 7 \times (9 - 7)$

$= 16 + (-21) \div 7 \times 2$

$= 16 + \left(\frac{-21}{7} \times 2\right)$

$= 16 + (-3 \times 2)$

$= 16 + (-6)$

$= 16 - 6$

$= 10$

(e) $8 \div [3 + (-15)] \div [(-2) \times 4 \times (-3)]$

$= 8 \div (3 - 15) \div [(-2) \times (-12)]$

$= 8 \div (-12) \div (24)$

$= \left(\frac{8}{-12}\right) \div 24$

$= \left(-\frac{2}{3}\right) \times \frac{1}{24}$

$= -\frac{1}{36}$

(f) $\{[(-23) - (-11)] \div 6 - 7 \div (-7)\} \times 1997$

$= [(-23 + 11) \div 6 - 7 \div (-7) \times 1997$

$= [(-12) \div 6 - 7 \div (-7)] \times 1997$

$= \left[\left(\frac{-12}{6}\right) - \left(\frac{7}{-7}\right)\right] \times 1997$

$= [(-2) - (-1)] \times 1997$

$= (-2 + 1) \times 1997$

$= (-1) \times 1997$

$= -1997$

21. (a) $\frac{(-2) \times (-5) + (-20)}{(-10)} = 1$

(b) $\frac{(-123) \times [19 + (-19)]}{38} = 0$

(c) $(-11) \times [-52 + (-17) - (-39)] = 330$

(d) $16 + (-21) \div 7 \times \{9 + [56 \div (-8)]\} = 10$

(e) $8 \div [3 + (-15)] \div [(-2) \times 4 \times (-3)] = -\frac{1}{36}$

(f) $\{[(-23) - (-11)] \div 6 - 7 \div (-7)\} \times 1997 = -1997$

22. (a) $-5\frac{2}{9} - 3\frac{1}{4} - 3\frac{5}{9}$

$= -5\frac{8}{36} - 3\frac{9}{36} - 3\frac{20}{36}$

$= (-5 - 3 - 3) - \frac{8}{36} - \frac{9}{36} - \frac{20}{36}$

$= -11 - \frac{(8 + 9 + 20)}{36}$

$= -11 - \frac{37}{36}$

$= -12\frac{1}{36}$

(b) $-3\frac{4}{5} - 1\frac{3}{10} - \left(-2\frac{3}{4}\right)$

$= -3\frac{16}{20} - 1\frac{6}{20} - \left(-2\frac{15}{20}\right)$

$= -3\frac{16}{20} - 1\frac{6}{20} + \left(2\frac{15}{20}\right)$

$= (-3 - 1 + 2) - \frac{16}{20} - \frac{6}{20} + \frac{15}{20}$

$= -2 + \frac{(-16 - 6 + 15)}{20}$

$= -2 + \left(\frac{-7}{20}\right)$

$= -2 - \frac{7}{20}$

$= -2\frac{7}{20}$

(c) $-2\frac{3}{4} + \left(-1\frac{1}{2}\right) - \left(-1\frac{2}{3}\right)$

$= -2\frac{3}{4} - 1\frac{1}{2} + 1\frac{2}{3}$

$= -2\frac{9}{12} - 1\frac{6}{12} + 1\frac{8}{12}$

$= (-2 - 1 + 1) - \frac{9}{12} - \frac{6}{12} + \frac{8}{12}$

$= -2 - \frac{7}{12}$

$= -2\frac{7}{12}$

(d) $-\left(-3\frac{5}{7}\right) + 1\frac{3}{5} - \left(-\frac{3}{7}\right)$

$= 3\frac{5}{7} + 1\frac{3}{5} + \frac{3}{7}$

$= 3\frac{25}{35} + 1\frac{21}{35} + \frac{15}{35}$

$= (3 + 1) + \frac{25}{35} + \frac{21}{35} + \frac{15}{35}$

$= 4 + \frac{61}{35}$

$= 4 + 1\frac{26}{35}$

$= 5\frac{26}{35}$

(e) $\left(-\frac{1}{5}+\frac{1}{3}\right)+\left[\frac{1}{10}+\left(-\frac{1}{5}\right)\right]+\left(-\frac{1}{25}\right)$

$=\left(-\frac{3}{15}+\frac{5}{15}\right)+\left[\frac{1}{10}+\left(-\frac{2}{10}\right)\right]+\left(-\frac{1}{25}\right)$

$=\frac{2}{15}+\left(-\frac{1}{10}\right)-\frac{1}{25}$

$=\frac{2}{15}-\frac{1}{10}-\frac{1}{25}$

$=\frac{20}{150}-\frac{15}{150}-\frac{6}{150}$

$=-\frac{1}{150}$

23. (a) $-5\frac{2}{9}-3\frac{1}{4}-3\frac{5}{9}=-12\frac{1}{36}$

(b) $-3\frac{4}{5}-1\frac{3}{10}-\left(-2\frac{3}{4}\right)=-2\frac{7}{20}$

(c) $-2\frac{3}{4}+\left(-1\frac{1}{2}\right)-\left(-1\frac{2}{3}\right)=-2\frac{7}{12}$

(d) $-\left(-3\frac{5}{7}\right)+1\frac{3}{5}-\left(-\frac{3}{7}\right)=5\frac{26}{35}$

(e) $\left(-\frac{1}{5}+\frac{1}{3}\right)+\left[\frac{1}{10}+\left(-\frac{1}{5}\right)\right]+\left(-\frac{1}{25}\right)=-\frac{1}{150}$

24. (a) $(-4)\div\left(-\frac{1}{4}\right)\times(-4)$

$=(-4)\times(-4)\times(-4)$

$=16\times(-4)$

$=-64$

(b) $\left(-2\frac{2}{5}\right)\times\left(\frac{5}{6}\right)\div(-13)$

$=\left(-\frac{\cancel{12}^{2}}{\cancel{5}_{1}}\right)\times\left(\frac{\cancel{5}^{1}}{\cancel{6}_{1}}\right)\div(-13)$

$=-2\div(-13)$

$=\frac{-2}{-13}$

$=\frac{2}{13}$

(c) $\left(1\frac{7}{15}\right)\div\left(-17\frac{2}{7}\right)\times\left(3\frac{3}{14}\right)$

$=\left(\frac{22}{15}\right)\div\left(\frac{-121}{7}\right)\times\left(\frac{45}{14}\right)$

$=\left(\frac{22}{\cancel{15}_{1}}\right)\times\left(-\frac{\cancel{7}^{1}}{121}\right)\times\left(\frac{\cancel{45}^{3}}{\cancel{14}_{2}}\right)$

$=-\frac{66}{242}$

$=-\frac{3}{11}$

(d) $\left(-2\frac{5}{7}\right)\div\left(1\frac{1}{3}\times\frac{3}{4}\right)$

$=\left(-2\frac{5}{7}\right)\div\left(\frac{4}{3}\times\frac{3}{4}\right)$

$=\left(-2\frac{5}{7}\right)\div 1$

$=-2\frac{5}{7}$

(e) $\left(3\frac{3}{5}\right)\times(-6)\div\left(-4\frac{4}{5}\right)$

$=\left(\frac{18}{5}\right)\times(-6)\div\left(-\frac{24}{5}\right)$

$=\left(\frac{18}{\cancel{5}_{1}}\right)\times(-\cancel{6}^{1})\times\left(-\frac{\cancel{5}^{1}}{\cancel{24}_{4}}\right)$

$=\frac{18}{4}$

$=4\frac{2}{4}$

$=4\frac{1}{2}$

(f) $\frac{1}{4}+\left(-\frac{3}{4}\right)\times\left(-1\frac{1}{4}\right)$

$=\frac{1}{4}+\left(-\frac{3}{4}\right)\times\left(-\frac{5}{4}\right)$

$=\frac{1}{4}+\left(\frac{15}{16}\right)$

$=\frac{4}{16}+\left(\frac{15}{16}\right)$

$=\frac{19}{16}$

$=1\frac{3}{16}$

(g) $\left[\left(-9\frac{1}{4}-\left(-7\frac{3}{5}\right)\right)\right]\div 2\frac{3}{4}$

$=\left[\left(-9\frac{1}{4}+7\frac{3}{5}\right)\right]\div 2\frac{3}{4}$

$=\left[\left(-9\frac{5}{20}+7\frac{12}{20}\right)\right]\div 2\frac{3}{4}$

$=\left[(-9+7)-\frac{5}{20}+\frac{12}{20}\right]\div 2\frac{3}{4}$

$=\left[(-2)+\frac{7}{20}\right]\div 2\frac{3}{4}$

$=\left(-\frac{33}{20}\right)\div 2\frac{3}{4}$

$=\left(-\frac{33}{20}\right)\div\frac{11}{4}$

$=\left(-\frac{33}{20}\right)\times\frac{4}{11}$

$=-\frac{3}{5}$

(h) $\left[\left(-1\frac{1}{4}\right)+1\frac{2}{5}\right] \div \left[(-6)-\frac{4}{7}\times\left(-2\frac{3}{4}\right)\right]$

$= \left[\left(-1\frac{5}{20}\right)+1\frac{8}{20}\right] \div \left[(-6)-\frac{4}{7}\times\left(-\frac{11}{4}\right)\right]$

$= \left[(-1+1)-\frac{5}{20}+\frac{8}{20}\right] \div \left[(-6)-\frac{4}{7}\times\left(-\frac{11}{4}\right)\right]$

$= \left(\frac{3}{20}\right) \div \left[(-6)-\left(-\frac{11}{7}\right)\right]$

$= \left(\frac{3}{20}\right) \div \left[\left(-\frac{42}{7}\right)+\left(\frac{11}{7}\right)\right]$

$= \frac{3}{20} \div \left(-\frac{31}{7}\right)$

$= \frac{3}{20} \times \left(-\frac{7}{31}\right)$

$= -\frac{21}{620}$

(i) $\left(-\frac{3}{4}\right) \times 1\frac{1}{2} + \left(-\frac{3}{4}\right) \times \left(-2\frac{1}{2}\right)$

$= \left(-\frac{3}{4}\right) \times \frac{3}{2} + \left(-\frac{3}{4}\right) \times \left(-\frac{5}{2}\right)$

$= \left(-\frac{9}{8}\right) + \frac{15}{8}$

$= \frac{6}{8}$

$= \frac{3}{4}$

25. (a) $(-4) \div \left(-\frac{1}{4}\right) \times (-4) = -64$

(b) $\left(-2\frac{2}{5}\right) \times \left(\frac{5}{6}\right) \div (-13) = \frac{2}{13}$

(c) $\left(1\frac{7}{15}\right) \div \left(-17\frac{2}{7}\right) \times \left(3\frac{3}{14}\right) = -\frac{3}{11}$

(d) $\left(-2\frac{5}{7}\right) \div \left(1\frac{1}{3} \times \frac{3}{4}\right) = -2\frac{5}{7}$

(e) $\left(3\frac{3}{5}\right) \times (-6) \div \left(-4\frac{4}{5}\right) = 4\frac{1}{2}$

(f) $\frac{1}{4} + \left(-\frac{3}{4}\right) \times \left(-1\frac{1}{4}\right) = 1\frac{3}{16}$

(g) $\left[\left(-9\frac{1}{4}-\left(-7\frac{3}{5}\right)\right)\right] \div 2\frac{3}{4} = -\frac{3}{5}$

(h) $\left[\left(-1\frac{1}{4}\right)+1\frac{2}{5}\right] \div \left[(-6)-\frac{4}{7}\times\left(-2\frac{3}{4}\right)\right] = -\frac{21}{620}$

(i) $\left(-\frac{3}{4}\right) \times 1\frac{1}{2} + \left(-\frac{3}{4}\right) \times \left(-2\frac{1}{2}\right) = \frac{3}{4}$

26. Altitude at which the plane is flying now
$= 650 - 150 + 830$
$= 500 + 830$
$= 1330$ m

27. Temperature of Singapore after rain stops
$= 24°C + 8°C - 12°C + 6°C$
$= 32°C - 12°C + 6°C$
$= 20°C + 6°C$
$= 26°C$

28. Let x be the number of boys.
Number of sweets each boy will have $= 6 - 1$
$= 5$
Since Ali took the last sweet,
total number of sweets = 41
$5x + 1 = 41$
$5x = 40$
$x = \frac{40}{5}$
$x = 8$
$\therefore$ 8 boys were seated around the table.

29.

Packet	1	2	3	4	5
Mass above or below the standard mass (g)	–28	–13	+10	–19	+5
Actual mass (g)	1000 – 28 = 972 g	1000 – 13 = 987 g	1000 + 10 = 1010 g	1000 – 19 = 981 g	1000 + 5 = 1005 g

Packet 5

(b) (i) Difference $= 1005 - 972$
$= 33$ g

(ii) Difference $= 1005 - 981$
$= 24$ g

(iii) Difference $= 1005 - 987$
$= 18$ g

Packet 5 and packet 1 have the largest difference.

(c) Mass of rice in packet 6
$= \frac{972 + 1010}{2}$
$= \frac{1982}{2}$
$= 991$ g

Chapter 3 Percentage

Basic

1. **(a)** $18\% = \frac{18}{100}$

$= \frac{9}{50}$

(b) $85\% = \frac{85}{100}$

$= \frac{17}{20}$

(c) $125\% = \frac{125}{100}$

$= 1\frac{1}{4}$

(d) $210\% = \frac{210}{100}$

$= 2\frac{1}{10}$

(e) $0.25\% = \frac{0.25}{100}$

$= \frac{0.25 \times 100}{100 \times 100}$

$= \frac{25}{10\,000}$

$= \frac{1}{400}$

(f) $4.8\% = \frac{4.8}{100}$

$= \frac{4.8 \times 10}{100 \times 10}$

$= \frac{48}{1000}$

$= \frac{6}{125}$

(g) $1\frac{1}{3}\% = \frac{4}{3}\%$

$= \frac{4}{3} \div 100$

$= \frac{4}{3} \times \frac{1}{100}$

$= \frac{4}{300}$

$= \frac{1}{75}$

(h) $12\frac{1}{2}\% = \frac{25}{2}\%$

$= \frac{25}{2} \div 100$

$= \frac{25}{2} \times \frac{1}{100}$

$= \frac{25}{200}$

$= \frac{1}{8}$

2. **(a)** $9\% = \frac{9}{100}$

$= 0.09$

(b) $99\% = \frac{99}{100}$

$= 0.99$

(c) $156\% = \frac{156}{100}$

$= 1.56$

(d) $0.05\% = \frac{0.05}{100}$

$= 0.0005$

(e) $0.68\% = \frac{0.68}{100}$

$= 0.0068$

(f) $1.002\% = \frac{1.002}{100}$

$= 0.010\,02$

(g) $2.4\% = \frac{2.4}{100}$

$= 0.024$

(h) $14\frac{2}{5}\% = \frac{72}{5}\%$

$= \frac{72}{5} \div 100$

$= \frac{72}{5} \times \frac{1}{100}$

$= \frac{72}{500}$

$= 0.144$

3. **(a)** $\frac{4}{625} = \frac{4}{625} \times 100\%$

$= 0.64\%$

(b) $\frac{9}{125} = \frac{9}{125} \times 100\%$

$= 7.2\%$

(c) $\frac{6}{25} = \frac{6}{25} \times 100\%$

$= 24\%$

(d) $\frac{3}{4} = \frac{3}{4} \times 100\%$

$= 75\%$

(e) $\frac{19}{20} = \frac{19}{20} \times 100\%$

$= 95\%$

(f) $\frac{9}{8} = \frac{9}{8} \times 100\%$

$= 112.5\%$

(g) $\frac{7}{5} = \frac{7}{5} \times 100\%$

$= 140\%$

(h) $\frac{33}{8} = \frac{33}{8} \times 100\%$

$= 412.5\%$

4. **(a)** $0.0034 = 0.0034 \times 100\%$

$= 0.34\%$

(b) $0.027 = 0.027 \times 100\%$

$= 2.7\%$

(c) $0.05 = 0.05 \times 100\%$

$= 5\%$

(d) $0.14 = 0.14 \times 100\%$

$= 14\%$

(e) $0.5218 = 0.5218 \times 100\%$

$= 52.18\%$

(f) $6.325 = 6.325 \times 100\%$

$= 632.5\%$

(g) $16.8 = 16.8 \times 100\%$

$= 1680\%$

(h) $332 = 332 \times 100$

$= 33\,200\%$

5. **(a)** Convert 1*l* to m*l*.

$1l = 1000$ m*l*

$\frac{175}{1000} \times 100\% = 17.5\%$

(b) Convert 1 day to hours.

1 day = 24 hours

$\frac{6}{24} \times 100\% = 25\%$

(c) Convert 1 hour to minutes.

1 hour = 60 minutes

$\frac{20}{60} \times 100\% = 33\frac{1}{3}\%$

(d) Convert PKR 1.44 to paisas.

PKR 1.44 = 144 paisas

$\frac{80}{144} \times 100\% = 55\frac{5}{9}\%$

(e) Convert 20 cm to mm.

20 cm = 20 × 10 = 200 mm

$\frac{225}{200} \times 100\% = 112.5\%$

(f) Convert 45 kg to g.

45 kg = 45 × 1000 = 45 000 g

$\frac{45\,000}{36\,000} \times 100\% = 125\%$

(g) Convert 2 years to months.

2 years = 2 × 12 = 24 months

$\frac{24}{18} \times 100\% = 133\frac{1}{3}\%$

(h) Convert PKR 4.40 to paisas.

PKR 4.40 = 440 paisas

$\frac{440}{99} \times 100\% = 444\frac{4}{9}\%$

6. Total amount of mixture = 8 + 42 = 50*l*

(i) Percentage of milk in the mixture

$= \frac{42}{50} \times 100\%$

$= 84\%$

(ii) Percentage of water in the mixture

$= \frac{8}{50} \times 100\%$

$= 16\%$

7. Percentage of latecomers in school *A*

$= \frac{25}{1500} \times 100\%$

$= 1\frac{2}{3}\%$ or 1.67% (to 3 s.f.)

Percentage of latecomers in school *B*

$= \frac{25}{1800} \times 100\%$

$= 1\frac{7}{18}\%$ or 1.39% (to 3 s.f.)

School *A* has 1.67% of students coming late whereas school *B* has 1.39% of students coming late. Thus, school *B* has a lower percentage of latecomers.

8. **(a)** 0.25% of 4000

$= \frac{0.25}{100} \times 4000$

$= 0.25 \times 40$

$= 10$

(b) 6% of 200 $= \frac{6}{100} \times 200$

$= 12$

(c) 7.5% of PKR 2500 $= \frac{7.5}{100} \times 2500$

$= 7.5 \times 25$

$=$ PKR 187.50

(d) 8% of 130 g $= \frac{8}{100} \times 130$

$= 10.4$ g

(e) 20.6% of 15 000 people

$= \frac{20.6}{100} \times 15\,000$

$= 20.6 \times 150$

$= 3090$ people

(f) $37\frac{1}{2}$ % of 56 cm

$= \frac{75}{2}$ % of 56 cm

$= \frac{75}{2} \times \frac{1}{100} \times 56$

$= 21$ cm

(g) 45% of 4 kg

$= \frac{45}{100} \times 4$

$= 1.8$ kg

(h) $66\frac{2}{3}$ % of 72 litres

$= \frac{200}{3}$ % of 72 litres

$= \frac{200}{3} \times \frac{1}{100} \times 72$

$= 48$ litres

(i) $112\frac{1}{2}$ % of 200 m

$= \frac{225}{2}$ % of 200 m

$= \frac{225}{2} \times \frac{1}{100} \times 200$

$= 225$ m

(j) 180% of 320

$= \frac{180}{100} \times 320$

$= 576$

9. Method 1

Number of kilograms of zinc = 25% of 60

$= \frac{25}{100} \times 60$

$= 15$

Number of kilograms of copper

$= 60 - 15$

$= 45$

The ingot of copper contains 45 kg of copper.

Method 2

Percentage of copper in ingot = 100% – 25% = 75%

Number of kilograms of copper in ingot = 75% of 60

$= \frac{75}{100} \times 60$

$= 45$

The ingot of brass contains 45 kg of copper.

10. (a) Required value = 110% of PKR 60

$= \frac{110}{100} \times 60$

= PKR 66

(b) Required value = 128% of 69 *l*

$= \frac{128}{100} \times 69$

$= 88.32$ *l*

(c) Required value = 225% of 50 m

$= \frac{225}{100} \times 50$

$= 112.5$ m

(d) Required value = 400% of 24 kg

$= \frac{400}{100} \times 24$

$= 96$ kg

(e) Required value = $112\frac{1}{2}$ % of 32 g

$= \frac{225}{2}$ % $\times$ 32

$= \left(\frac{225}{2} \div 100\right) \times 32$

$= \frac{225}{2} \times \frac{1}{100} \times 32$

$= 36$ g

(f) Required value = 100.03% of PKR 400

$= \frac{100.03}{100} \times 400$

$= 100.03 \times 4$

= PKR 400.12

(g) Required value = 100.5% of PKR 4000

$= \frac{100.5}{100} \times 4000$

$= 100.5 \times 40$

= PKR 4020

(h) Required value = 2600% of PKR 1.50

$= \frac{2600}{100} \times 1.50$

= PKR 39

11. (a) Required value
$= 99.4\%$ of 1.25 km
$= \frac{99.4}{100} \times 1.25$
$= 1.2425$ km

(b) Required value
$= 95\%$ of PKR 88
$= \frac{95}{100} \times 88$
$=$ PKR 83.60

(c) Required value
$= 93\%$ of PKR 7500
$= \frac{93}{100} \times 7500$
$=$ PKR 6975

(d) Required value
$= 87\frac{1}{2}\%$ of 64 g
$= \frac{175}{2}\%$ of 64 g
$= \frac{175}{2} \times \frac{1}{100} \times 64$
$= 56$ g

(e) Required value
$= 86.5\%$ of 78 kg
$= \frac{86.5}{100} \times 78$
$= 67.47$ kg

(f) Required value
$= 85\%$ of 124 l
$= \frac{85}{100} \times 124$
$= 105.4\ l$

(g) Required value
$= 58\%$ of 350 m^2
$= \frac{58}{100} \times 350$
$= 203\ m^2$

(h) Required value
$= 15\%$ of 520
$= \frac{15}{100} \times 520$
$= 78$

12. (a) Let the number be x.
12% of $x = 48$
$\frac{12}{100} \times x = 48$
$x = 48 \div \frac{12}{100}$
$= 48 \times \frac{100}{12}$
$x = 400$

(b) Let the number be x.
$15\frac{5}{8}\%$ of $x = 555$
$\frac{125}{8}\% \times x = 555$
$\left(\frac{125}{8} \div 100\right) \times x = 555$
$\frac{125}{8} \times \frac{1}{100} \times x = 555$
$\frac{5}{32} \times x = 555$
$x = 555 \div \frac{5}{32}$
$= 3552$

(c) Let the number be x.
21% of $x = 147$
$\frac{21}{100} \times x = 147$
$x = 147 \div \frac{21}{100}$
$= 147 \times \frac{100}{21}$
$x = 700$

(d) Let the number be x.
77.5% of $x = 217$
$\frac{77.5}{100} \times x = 217$
$x = 217 \div \frac{77.5}{100}$
$= 217 \times \frac{100}{77.5}$
$= 280$

(e) Let the number be x.
124% of $x = 155$
$\frac{124}{100} \times x = 155$
$x = 155 \div \frac{124}{100}$
$= 155 \times \frac{100}{124}$
$= 125$

13. (a) Let the number be x.

120% of $x = 48$

$$\frac{120}{100} \times x = 48$$

$$x = 48 \div \frac{120}{100}$$

$$= 48 \times \frac{100}{120}$$

$$= 40$$

(b) Let the number be x.

70% of $x = 147$

$$\frac{70}{100} \times x = 147$$

$$x = 147 \div \frac{70}{100}$$

$$= 147 \times \frac{100}{70}$$

$$= 210$$

(c) Let the number be x.

$$33\frac{1}{3}\% \text{ of } x = 432$$

$$\frac{100}{3}\% \text{ of } x = 432$$

$$\left(\frac{100}{3} \div 100\right) \times x = 432$$

$$\frac{100}{3} \times \frac{1}{100} \times x = 432$$

$$\frac{1}{3}x = 432$$

$$x = 432 \div \frac{1}{3}$$

$$= 1296$$

14. Increase in the number of buses operating

$= 1420 - 1000$

$= 420$

Percentage increase in the number of buses in operation

$$= \frac{\text{Increase}}{\text{Original value}} \times 100\%$$

$$= \frac{420}{1000} \times 100\%$$

$= 42\%$

15. Decrease in the price of MP3 player

$=$ PKR $382 -$ PKR 261.50

$=$ PKR 120.50

Percentage decrease in the price

$$= \frac{\text{Decrease}}{\text{Original value}} \times 100\%$$

$$= \frac{120.5}{382} \times 100\%$$

$= 31.5\%$ (to 3 s.f.)

16. 120% of Jamal's income $=$ PKR 120

$$\frac{120}{100} \times \text{Jamal's income} = \text{PKR } 120$$

$$\text{Jamal's income} = 120 \div \frac{120}{100}$$

$$= 120 \times \frac{100}{120}$$

$=$ PKR 100

17. Price of notebook in 2013 $= 70\%$ of PKR 2000

$$= \frac{70}{100} \times 2000$$

$=$ PKR 1400

Price of notebook in 2014 $= 70\%$ of PKR 1400

$$= \frac{70}{100} \times 1400$$

$=$ PKR 980

Intermediate

18. Let the total number of students taking Additional Mathematics be x.

35% of $x = 42$

$$\frac{35}{100} \times x = 42$$

$$x = 42 \div \frac{35}{100}$$

$$= 42 \times \frac{100}{35}$$

$= 120$

Number of students taking Additional Mathematics in class C

$= 120 - 42 - 40$

$= 38$

19. Percentage of candidates who obtained grade C

$= 100\% - 18\% - 38\%$

$= 44\%$

Let the total number of candidates be x.

44% of $x = 77$

$$\frac{44}{100} \times x = 77$$

$$x = 77 \div \frac{44}{100}$$

$$= 77 \times \frac{100}{44}$$

$= 175$

The total number of candidates is 175.

20. Amount of milk in the solution
= 30% of 125 l
$= \frac{30}{100} \times 125$
= 37.5 l
Let the amount of water to be added be x l.
$\frac{37.5}{125 + x} = 14\%$
$\frac{37.5}{125 + x} = \frac{7}{50}$
$875 + 7x = 1875$
$7x = 100$
$x = 142\frac{6}{7}$
Amount of water added = $142\frac{6}{7}$ l

21. 140% of price in first half of 2013 = PKR 52 640
Price in first half of 2013 = 52 640 ÷ $\frac{140}{100}$
= 52 640 × $\frac{100}{140}$
= PKR 37 600
98% of price in 2012 = PKR 37 600
Price in 2012 = 37 600 ÷ $\frac{98}{100}$
= 37 600 × $\frac{100}{98}$
= PKR 38 367.35
105% of original price of painting = PKR 38 367.35
Original price of painting
= 38 367.35 ÷ $\frac{105}{100}$
= 38 367.35 × $\frac{100}{105}$
= PKR 36 540.33 (to the nearest paisa)

22. Number of girls in the club = 70% of 40
$= \frac{70}{100} \times 40$
= 28
Number of boys in the club = 40 – 28
= 12
Let the number of new members who are girls be x and the number of new members who are boys be y.
Then $y - x = 6$.
$y = 6 + x$
New percentage of girls in the club = 60%
$\frac{28 + x}{40 + x + y} = \frac{60}{100}$
$\frac{28 + x}{40 + x + y} = \frac{3}{5}$
$5(28 + x) = 3(40 + x + y)$
$140 + 5x = 120 + 3x + 3y$
$140 - 120 + 5x - 3x = 3y$
$20 + 2x = 3y$
Substitute $y = 6 + x$:
$20 + 2x = 3(6 + x)$
$20 + 2x = 18 + 3x$
$20 - 18 = 3x - 2x$
$x = 2$
$y = 6 + 2$
= 8
No. of members who are boys = 12 + 8
= 20

23. (i) 3 parts of the length AB = 3 cm
1 part of the length AB = 1 cm
7 parts, which is the length of AB = 7 cm
(ii) BC = 135% of AB
$= \frac{135}{100} \times 7$
= 9.45 cm
(iii) AC = 85% of BC
$= \frac{85}{100} \times 9.45$
= 8.0325 cm

24. (i) Selling price of the flat = 115% of PKR 145 000
$= \frac{115}{100} \times 145\,000$
= PKR 166 750
Amount gained by selling the flat
= 166 750 – 145 000
= PKR 21 750
(ii) Selling price of the car = 88% of PKR 50 000
$= \frac{88}{100} \times 50\,000$
= PKR 44 000
Amount lost by selling his car
= 50 000 – 44 000
= PKR 6000
(iii) Yes, he still gained an amount of
PKR 21 750 – PKR 6000 = PKR 15 750

Advanced

25. (a) Zhi Xiang's new monthly salary under scheme B
= 104.5% of PKR 1500 + PKR 50
$= \frac{104.5}{100} \times 1500 + 50$
= 1567.5 + 50
= PKR 1617.50
Zhi Xiang's new salary as a percentage of his present salary
$= \frac{1617.50}{1500} \times 100\%$
= 108% (to 3 s.f.)

(b) Tom's new monthly salary under scheme *A*
= 106% of PKR 1200
$= \frac{106}{100} \times 1200$
= PKR 1272
Tom's new monthly salary under scheme *B*
= 104.5% of PKR 1200 + PKR 50
$= \frac{104.5}{100} \times 1200 + 50$
= PKR 1304
∴ Since Tom's salary will be higher under scheme *B*, he should choose scheme *B*.

(c) Let Sharon's current monthly wage be PKR x.
106% of PKR x = 104.5% of PKR x + PKR 50
(106 – 104.5)% of x = 50
1.5% of x = 50
$\frac{1.5}{100} \times x = 50$
$x = 50 \div \frac{1.5}{100}$
$= 50 \times \frac{100}{1.5}$
x = 3333.33 (to the nearest paisa)
∴ Sharon's salary is PKR 3333.33.

New Trend

26. Let x be the total number of crayons.
Number of blue crayons = $\frac{4}{9}x$
Number of red crayons = $65\% \times \frac{5}{9}x$
$= \frac{65}{100} \times \frac{5}{9}x$
$= \frac{13}{36}x$
Number of yellow crayons = $x - \frac{4}{9}x - \frac{13}{36}x$
$= \frac{7}{36}x$
$\frac{7}{36}x = 14$
$x = 72$
There are 72 crayons altogether.

27. In 2013, the value of the bracelet
= 110% of PKR 12 650
$= \frac{110}{100} \times 12\,650$
= PKR 13 915
In 2014, the value of the bracelet
= 110% of PKR 13 915
$= \frac{110}{100} \times 13\,915$
= PKR 15 306.50
In 2015, the value of the bracelet
= 110% of PKR 15 306.50
$= \frac{110}{100} \times 15\,306.50$
= PKR 16 837.15
$\frac{16\,837.15 - 12\,650}{12\,650} \times 100\% = 33.1\%$
The value of the bracelet in 2015 is PKR 16 837.15 and the overall percentage increase is 33.1%.

28. 103% of original bill = PKR 82.70
$\frac{103}{100} \times$ original bill = 82.70
Original bill = $82.70 \div \frac{103}{100}$
$= 82.70 \times \frac{100}{103}$
= PKR 80.29

29. (a) Convert 3.96 m to cm.
3.96 m = (3.96 × 100) cm
= 396 cm
$\frac{33}{396} \times 100\% = 8\frac{1}{3}\%$

(b) 15 ÷ 0.3 = 50
50 glasses can be filled.

Chapter 4 Sets

Basic

1. **(a)** Yes, because it is clear if a pupil has no siblings.

(b) No, because a bag may be considered nice by some but not by other.

(c) No, because a singer may be considered attractive to some, but not to others.

(d) No, because a song may be well-liked by some, but not others.

(e) Yes, because it is clear whether a teacher teaches Art.

(f) No, because a movie may be considered funny to some, but not others.

2. **(a)** T

(b) T

Intermediate

3. $\xi = \{x : x \text{ is an integer}, 0 \leqslant x < 25\}$

$= \{0, 1, 2, 3, \ldots, 23, 24\}$

$B = \{x : x \text{ is divisible by } 5\} = \{0, 5, 10, 15, 20\}$

$C = \{x : x \text{ is prime and } x \leqslant 19\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19\}$

4. $\xi = \{x : x \text{ is an integer}, 0 < x \leqslant 13\}$

$= \{1, 2, 3, \ldots, 11, 12, 13\}$

$A = \{x : 2x > 9\}$

$B = \{x : (x - 2)(x - 5) = 0\}$

$C = \{x : x \text{ is prime}\}$

(a) $C = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

(b) $C = \{2, 5\}$

(c) $C = \{1, 3, 5, 7, 11, 13\}$

$A \cap C = \{5, 7, 11, 13\}$

Chapter 5 Number Pattern and Algebraic Manipulation

Basic

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are 26 and 31.

(b) Rule: Subtract 3 from each term to get the next term. The next two terms are 19 and 16.

(c) Rule: Multiply each term by 10 to get the next term. The next two terms are 10 000 and 100 000.

(d) Rule: Multiply each term by 5 to get the next term. The next two terms are 250 and 1250.

(e) Rule: Multiply the previous term by the term number to get the next term. The next two terms are $24 \times 5 = 120$ and $120 \times 6 = 720$.

(f) Rule: Take the cube of each term number to get the next term. The next two terms are $5^3 = 125$ and $6^3 = 216$.

(g) Rule: Subtract 5 from each term to get the next term. The next two terms are 32 and 27.

(h) Rule: Denote $64 = 8^2$ as the first term. Subtract 1 from the base of each term and square it to get the next term. The next two terms are $4^2 = 16$ and $3^2 = 9$.

(i) Rule: Add the previous term by its term number to get the next term. The next two terms are $12 + 5 = 17$ and $17 + 6 = 23$.

(j) Rule: Add the square of the term number to each term to get the next term. The next two terms are $34 + 5^2 = 59$ and $59 + 6^2 = 95$.

(k) Rule: Add the term number to the previous term to get the next term. The next two terms are $30 + 5 = 35$ and $35 + 6 = 41$.

(l) Denote 7 as the zero term.
Rule: Add each term by 2 to the power of its term number to get the next term. The next two terms are $22 + 2^4 = 38$ and $38 + 2^5 = 70$.

(m) Denote 90 as the first term.
Rule 1: Subtract 10 from each odd term to get the next odd term.
Rule 2: Add 10 to each even term to get the next even term. The next two terms are 60 and 40.

(n) Rule: Denote $1024 = 2^{10}$ as the first term. Subtract 1 from the power of each term to get the next term. The next two terms are $2^5 = 32$ and $2^4 = 16$.

2. **(i)** The next three terms of the sequence are 48, 96 and 192.

(ii) The next three terms of the sequence are 52, 100, 196.
Add 4 to the sequence in part **(i)**.

3. **(a)** $(2x + 5y) - 4 = 2x + 5y - 4$

(b) $(3x)(7y) + 9z = 21xy + 9z$

(c) $(7x)(11y) \times 2z = 77xy \times 2z$
$= 154xyz$

(d) $(3z + 7s) \div 5a = \dfrac{3z + 7s}{5a}$

(e) $r^3 - (p \div 3q) = r^3 - \dfrac{p}{3q}$

(f) $3w \div (3x + 7y) = \dfrac{3w}{3x + 7y}$

(g) $(k \div 2y) - 9(x)(3h) = \dfrac{k}{2y} - 27xh$

4. **(a)** $7b - 3c + 4a$
$= 7(2) - (3)(-1) + 4(3)$
$= 14 + 3 + 12$
$= 29$

(b) $(5b)^2$
$= (5 \times 2)^2$
$= (10)^2$
$= 100$

(c) $(2a + b + c)(5b - 3a)$
$= (2 \times 3 + 2 + (-1))(5 \times 2 - 3 \times 3)$
$= (7)(1)$
$= 7$

(d) $(a - b)^2 - (b - c)^2$
$= (3 - 2)^2 - (2 - (-1))^2$
$= 1^2 - (3)^2$
$= -8$

(e) $2a^2 - 3b^2 + 3abc$
$= 2(3)^2 - 3(2)^2 + 3(3)(2)(-1)$
$= 18 - 12 - 18$
$= -12$

(f) $a^b - c^a + b^c$
$= (3)^2 - (-1)^3 + (2)^{(-1)}$
$= 9 + 1 + \dfrac{1}{2}$
$= 10\dfrac{1}{2}$

(g) $\dfrac{a}{b} - \dfrac{b}{c}$
$= \dfrac{3}{2} - \dfrac{2}{-1}$
$= 1\dfrac{1}{2} + 2$
$= 3\dfrac{1}{2}$

(h) $\dfrac{8b-(3a)^2}{c}$

$= \dfrac{8(2)-(3\times 3)^2}{(-1)}$

$= \dfrac{16-9^2}{-1}$

$= \dfrac{16-81}{-1}$

$= 65$

(i) $\dfrac{b+c}{a} + \dfrac{a+bc}{b}$

$= \dfrac{2+(-1)}{3} + \dfrac{3+(2)(-1)}{2}$

$= \dfrac{1}{3} + \dfrac{1}{2}$

$= \dfrac{5}{6}$

5. **(a)** $3x + 9y + (-11y)$

$= 3x + 9y - 11y$

$= 3x - 2y$

(b) $-a - 3b + 7a - 10b$

$= 7a - a - 3b - 10b$

$= 6a - 13b$

(c) $13d + 5c + (-13c + 5d)$

$= 13d + 5c - 13c + 5d$

$= 13d + 5d + 5c - 13c$

$= 18d - 8c$

$= -8c + 18d$

(d) $7pq - 11hk + (-3pq - 21kh)$

$= 7pq - 11hk - 3pq - 21kh$

$= 4pq - 32hk$

6. **(a)** $5x + 7y - 2x - 4y$

$= 5x - 2x + 7y - 4y$

$= 3x + 3y$

(b) $-3a - 7b + 11a + 11b$

$= -3a + 11a - 7b + 11b$

$= 11a - 3a + 11b - 7b$

$= 8a + 4b$

(c) $5u - 7v - 7u - 9v$

$= 5u - 7u - 7v - 9v$

$= -2u - 16v$

(d) $5p + 4q - 7r - 5q + 4p$

$= 5p + 4p + 4q - 5q - 7r$

$= 9p - q - 7r$

(e) $5pq - 7qp + 21 - 7$

$= -2pq + 14$

(f) $15x + 9y + 5x - 3y - 13$

$= 15x + 5x + 9y - 3y - 13$

$= 20x + 6y - 13$

(g) $8ab - 5bc + 21ba - 7cb$

$= 8ab + 21ab - 5bc - 7cb$

$= 29ab - 12bc$

(h) $-7 + mn + 9mn - 3mn - 25$

$= mn + 9mn - 3mn - 25 - 7$

$= 7mn - 32$

(i) $3h - 4gh + \dfrac{2}{3}h - \dfrac{1}{3}gh$

$= 3h + \dfrac{2}{3}h - 4gh - \dfrac{1}{3}gh$

$= 3\dfrac{2}{3}h - 4\dfrac{1}{3}gh$

(j) $\dfrac{3}{5}x - \dfrac{2}{3}xy + \dfrac{1}{4}x - \dfrac{1}{5}xy$

$= \dfrac{3}{5}x + \dfrac{1}{4}x - \dfrac{2}{3}xy - \dfrac{1}{5}xy$

$= \dfrac{17}{20}x - \dfrac{13}{15}xy$

7. **(a)** $3(3x - 5)$

$= 9x - 15$

(b) $7(5 - 7x)$

$= 35 - 49x$

(c) $11(4x + 5y)$

$= 44x + 55y$

(d) $-3(9k - 2)$

$= -27k + 6$

(e) $-7(-3h - 5)$

$= 21h + 35$

(f) $4(3a - 2b + c)$

$= 12a - 8b + 4c$

(g) $-5\left(\dfrac{1}{4}p - \dfrac{2}{5}q + \dfrac{1}{2}r\right)$

$= -\dfrac{5}{4}p + 2q - \dfrac{5}{2}r$

(h) $-\dfrac{1}{4}(8a - 5b + 3c)$

$= -2a + \dfrac{5}{4}b - \dfrac{3}{4}c$

8. **(a)** $5a - 3(2p + 3)$

$= 5a - 6p - 9$

(b) $3x - 5(x - y)$

$= 3x - 5x + 5y$

$= -2x + 5y$

(c) $5(a + 4) + 7(b - 2)$

$= 5a + 20 + 7b - 14$

$= 5a + 7b + 20 - 14$

$= 5a + 7b + 6$

(d) $3(2p - 3q) - 5(3p - 5q)$
$= 6p - 9q - 15p + 25q$
$= 6p - 15p + 25q - 9q$
$= -9p + 16q$

(e) $r(3x - y) - 3r(x - 7y)$
$= 3xr - ry - 3xr + 21ry$
$= 3xr - 3xr + 21ry - ry$
$= 20ry$

(f) $3(x + y + z) + 5y - 4z$
$= 3x + 3y + 3z + 5y - 4z$
$= 3x + 3y + 5y + 3z - 4z$
$= 3x + 8y - z$

9. In 4 years' time,
Rizwan will be $(x + 4)$ years old.
$\therefore$ His brother will be $3(x + 4)$ years old.

10. Let the largest odd integer be x.
Then the previous odd integer will be $(x - 2)$.
The smallest odd integer is $(x - 2) - 2 = x - 4$.
Sum of three consecutive odd integers
$= x + (x - 2) + (x - 4)$
$= x + x - 2 + x - 4$
$= x + x + x - 2 - 4$
$= 3x - 6$

11. (a) $\frac{1}{3}x + \frac{1}{5}y - \frac{1}{9}x - \frac{1}{15}y$
$= \frac{1}{3}x - \frac{1}{9}x + \frac{1}{5}y - \frac{1}{15}y$
$= \frac{3}{9}x - \frac{1}{9}x + \frac{3}{15}y - \frac{1}{15}y$
$= \frac{2}{9}y + \frac{2}{15}y$

(b) $\frac{3}{4}a - \frac{1}{5}b + 3a - \frac{4}{7}b$
$= \frac{3}{4}a + 3a - \frac{4}{7}b - \frac{1}{5}b$
$= 3\frac{3}{4}a - \frac{20}{35}b - \frac{7}{35}b$
$= 3\frac{3}{4}a - \frac{27}{35}b$

(c) $\frac{5}{6}c + \frac{8}{7}d - \frac{2}{9}c - \frac{5}{3}d$
$= \frac{5}{6}c - \frac{2}{9}c + \frac{8}{7}d - \frac{5}{3}d$
$= \frac{15}{18}c - \frac{4}{18}c + \frac{24}{21}d - \frac{35}{21}d$
$= \frac{11}{18}c - \frac{11}{21}d$

(d) $5f - \frac{5}{7}h + \frac{7}{8}k - \frac{4}{3}f - \frac{4}{5}h + \frac{12}{11}k$
$= 5f - \frac{4}{3}f - \frac{5}{7}h - \frac{4}{5}h + \frac{12}{11}k + \frac{7}{8}k$
$= 3\frac{2}{3}f - \frac{25}{35}h - \frac{28}{35}h + \frac{96}{88}k + \frac{77}{88}k$
$= 3\frac{2}{3}f - 1\frac{18}{35}h + 1\frac{85}{88}k$

Intermediate

12. (a) 18, 24
(b) 9, 16
(c) 250, 50
(d) 16, 23
(e) 3, 5
(f) $\frac{16}{17}, \frac{22}{23}$
(g) $\frac{17}{1}, \frac{1}{23}$

13. (a) The next three terms are 39, 51 and 65.
(b) The prime numbers are 11 and 29.
(c) For the two numbers to have HCF as 13, the two numbers must have a common factor 13 and the other factor less than 13. The other factor must be different for the two numbers.
The possible numbers are 13, 26, 39, 52, 65, …
$\therefore$ The two numbers whose HCF is 13 from this sequence are 39 and 65.
(d) By prime factorisation, $195 = 3 \times 5 \times 13$.
Thus the 3 numbers whose LCM is 195 may be 3×5, 3×13 and 5×13.
$\therefore$ The three numbers whose LCM is 195 from this sequence are 15, 39 and 65.

14. (a)

$k + 8$	Add 8 to a number k
$5(k + 8)$	Multiply the sum by 5
$5(k + 8) - (2k - 1)$	Subtract $(2k - 1)$ from the result

$= 5k + 40 - 2k + 1$
$= 5k - 2k + 40 + 1$
$= 3k + 41$

(b) Cost of 7 pencils
$= 7 \times p$
$= 7p$ paisas
Change after buying the pencils
= PKR 4.20
= 420 paisas
Amount Kiran had before buying the pencils
$= (420 + 7p)$ paisas

(c) Cost price of the apples
$= x(y + 3)$ paisas
Selling price of the apples
$= x(2y - 5)$ paisas
Profit
= selling price – cost price
$= x(2y - 5) - x(y + 3)$
$= 2xy - 5x - xy - 3x$
$= 2xy - xy - 5x - 3x$
$= (xy - 8x)$ paisas

(d) Cost price of the microchips
$= (n)(2x)$
= PKR $2nx$
Selling price of the microchips
$= (n)(n - x)$
= PKR $n(n - x)$
Loss
= cost price – selling price
$= 2nx - n(n - x)$
$= 2nx - n^2 + nx$
= PKR $(3nx - n^2)$

15. (a) When $a = 4$, $m = -2$ and $n = -1$,
$4(-2)^2 - 3(4) - 5(-1)$
$= 16 - 12 + 5$
$= 4 + 5$
$= 9$

(b) When $a = 4$, $m = -2$ and $n = -1$,
$7(-1) + 3\frac{3}{4}(4) - (-2 - 4)$
$= -7 + 15 - (-6)$
$= 8 + 6$
$= 14$

16. (a) When $a = 2$, $c = -1$, $d = 5$ and $e = -4$,
$(2) - (-1)(5 - (-4))$
$= 2 + (5 + 4)$
$= 2 + 5 + 4$
$= 11$

(b) When $a = 2$, $c = -1$, $d = 5$ and $e = -4$,
$$\frac{2(-4) - 2}{(-1)^2 - 5(-4)}$$
$$= \frac{-8 - 2}{1 + 20}$$
$$= \frac{10}{21}$$

17. When $a = 2$, $b = -1$, $c = 0$ and $d = \frac{1}{2}$

(a) $(2a - b)^2$
$= (2 \times 2 - (-1))^2$
$= (4 + 1)^2$
$= 5^2$
$= 25$

(b) $(3a - b)(2c + d)$
$$= [3(2) - (-1)]\left[2(0) + \frac{1}{2}\right]$$
$$= (6 + 1)\left(\frac{1}{2}\right)$$
$$= \frac{7}{2}$$
$$= 3\frac{1}{2}$$

(c) $(5a - b)(2c + d) - b(ab + bc - 4cd)$
$$[5(2) - (-1)]\left[2(0) + \frac{1}{2}\right]$$
$$- (-1)\left[(2)(-1) + (-1)(0) - 4(0)\left(\frac{1}{2}\right)\right]$$
$$= (10 + 1)\left(\frac{1}{2}\right) + (-2)$$
$$= 3\frac{1}{2}$$

18. When $x = -3$,
$(2x - 1)(2x + 1)(2x + 3)$
$= (2(-3) - 1)(2(-3) + 1)(2(-3) + 3)$
$= (-6 - 1)(-6 + 1)(-6 + 3)$
$= (-7)(-5)(-3)$
$= -105$

19. When $x = -2$,
$$\frac{(-2) + 1}{(-2) - 1} + \frac{2(-2) - 1}{2(-2) + 1}$$
$$= \frac{-1}{-3} + \left(\frac{-5}{-3}\right)$$
$$= \frac{1}{3} + \frac{5}{3}$$
$= 2$

20. When $x = -2$,
$$\frac{(-2) - 5}{(-2) + 7} - 3(-2)^2$$
$$= \frac{-7}{5} - 12$$
$$= -13\frac{2}{5}$$

21. When $y = -3$,

$$\frac{x + 5(-3)}{5x - 7(-3)} = \frac{1}{4}$$

$$\frac{x - 15}{5x + 21} = \frac{1}{4}$$

$$4(x - 15) = 5x + 21$$

$$4x - 60 = 5x + 21$$

$$5x - 4x = -60 - 21$$

$$x = -81$$

22. (a) $a + b + c + (2b - c) + (3c + a)$

$= a + b + c + 2b - c + 3c + a$

$= a + a + b + 2b + c - c + 3c$

$= 2a + 3b + 3c$

(b) $2ab + 3bc + (5ac - 5ba) + (2cb + 5ab)$

$= 2ab + 3bc + 5ac - 5ba + 2cb + 5ab$

$= 2ab + 5ab - 5ba + 3bc + 2cb + 5ac$

$= 2ab + 5ab - 5ab + 3bc + 2bc + 5ac$

$= 2ab + 5bc + 5ac$

(c) $\frac{1}{2}xy + \left(\frac{1}{3}xy^2 - \frac{1}{4}yx\right) + \left(\frac{1}{6}xy^2 + xy\right)$

$= \frac{1}{2}xy + \frac{1}{3}xy^2 - \frac{1}{4}yx + \frac{1}{6}xy^2 + xy$

$= \frac{1}{2}xy - \frac{1}{4}yx + xy + \frac{1}{3}xy^2 + \frac{1}{6}xy^2$

$= \frac{2}{4}xy - \frac{1}{4}xy + xy + \frac{2}{6}xy^2 + \frac{1}{6}xy^2$

$= 1\frac{1}{4}xy + \frac{1}{2}xy^2$

(d) $a^2 + b^2 - c^2 + (2c^2 - b^2 + a^2) + (5a^2 + 7c^2)$

$= a^2 + b^2 - c^2 + 2c^2 - b^2 + a^2 + 5a^2 + 7c^2$

$= a^2 + a^2 + 5a^2 + b^2 - b^2 - c^2 + 2c^2 + 7c^2$

$= 7a^2 + 8c^2$

(e) $5abc - 7cb + 4ac + (4cba - 4bc + 3ca)$

$= 5abc - 7cb + 4ac + 4cba - 4bc + 3ca$

$= 5abc + 4cba + 4ac + 3ca - 7cb - 4bc$

$= 5abc + 4abc + 4ac + 3ac - 7bc - 4bc$

$= 9abc + 7ac - 11bc$

23. (a) $5(2x - 7y) - 4(y - 3x)$

$= 10x - 35y - 4y + 12x$

$= 10x + 12x - 35y - 4y$

$= 22x - 39y$

(b) $3a + 5ac - 2c - 4c - 6a - 8ca$

$= 3a - 6a + 5ac - 8ca - 2c - 4c$

$= -3a - 3ac - 6c$

(c) $5p + 3q - 4r - (6q - 3p + r)$

$= 5p + 3q - 4r - 6q + 3p - r$

$= 5p + 3p + 3q - 6q - 4r - r$

$= 8p - 3q - 5r$

(d) $3a^2 + 5a - 2(a - 2a^2)$

$= 3a^2 + 5a - 2a + 4a^2$

$= 3a^2 + 4a^2 + 5a - 2a$

$= 7a^2 + 3a$

(e) $2(x^2 - 5x) - 7(x - x^3 + x^2 - 1)$

$= 2x^2 - 10x - 7x + 7x^3 - 7x^2 + 7$

$= 7x^3 + 2x^2 - 7x^2 - 10x - 7x + 7$

$= 7x^3 - 5x^2 - 17x + 7$

(f) $7m - 2[6m - (3m - 4p)]$

$= 7m - 2[6m - 3m + 4p]$

$= 7m - 12m + 6m - 8p$

$= m - 8p$

(g) $7x - \{3x - [4x - 2(x + 3y)]\}$

$= 7x - \{3x - [4x - 2x - 6y]\}$

$= 7x - \{3x - [2x - 6y]\}$

$= 7x - \{3x - 2x + 6y\}$

$= 7x - \{x + 6y\}$

$= 7x - x - 6y$

$= 6x - 6y$

(h) $8a - \{2a - [3c - 6(a - 2c)]\}$

$= 8a - \{2a - [3c - 6a + 12c]\}$

$= 8a - \{2a - [3c + 12c - 6a]\}$

$= 8a - \{2a - [15c - 6a]\}$

$= 8a - \{2a - 15c + 6a\}$

$= 8a - \{2a + 6a - 15c\}$

$= 8a - \{8a - 15c\}$

$= 8a - 8a + 15c$

$= 15c$

(i) $12a - 3\{a - 4[c - 5(a - c)]$

$= 12a - 3\{a - 4[c - 5a + 5c]\}$

$= 12a - 3\{a - 4[c + 5c - 5a]\}$

$= 12a - 3\{a - 4[6c - 5a]\}$

$= 12a - 3\{a - 24c + 20a\}$

$= 12a - 3\{a + 20a - 24c\}$

$= 12a - 3\{21a - 24c\}$

$= 12a - 63a + 72c$

$= 72c - 51a$

(j) $7a^2 - 4a - 5a(a - 3) + 4(a - 5)$

$= 7a^2 - 4a - 5a^2 + 15a + 4a - 20$

$= 7a^2 - 5a^2 - 4a + 15a + 4a - 20$

$= 2a^2 + 15a - 20$

(k) $2a - 5(3ab - 4b) - 2(a - 2ba)$

$= 2a - 15ab + 20b - 2a + 4ab$

$= 2a - 2a - 15ab + 4ab + 20b$

$= -11ab + 20b$

$= 20b - 11ab$

(l) $4(x - 5y) - 5(2y - 3x) - (2x - 5y)$
$= 4x - 20y - 10y + 15x - 2x + 5y$
$= 4x + 15x - 2x - 20y - 10y + 5y$
$= 17x - 25y$

(m) $2(3x + y) - 5[3(x - 3y) - 4(2x - y)]$
$= 2(3x + y) - 5[3x - 9y - 8x + 4y]$
$= 2(3x + y) - 5[3x - 8x - 9y + 4y]$
$= 2(3x + y) - 5[- 5x - 5y]$
$= 6x + 2y + 25x + 25y$
$= 6x + 25x + 2y + 25y$
$= 31x + 27y$

(n) $\frac{1}{2}\left[14x - \frac{2}{3}(9x - 21y) - 2(x + y)\right]$
$= \frac{1}{2}[14x - 6x + 14y - 2x - 2y]$
$= \frac{1}{2}[14x - 6x - 2x + 14y - 2y]$
$= \frac{1}{2}[6x + 12y]$
$= 3x + 6y$

24. (a) $3a - 2b - 11 - (10a + 5b - 7)$
$= 3a - 2b - 11 - 10a - 5b + 7$
$= 3a - 10a - 5b - 2b - 11 + 7$
$= -7a - 7b - 4$

(b) $4x - 2z + 7 - (x - 3y - 5z + 5)$
$= 4x - 2z + 7 - x + 3y + 5z - 5$
$= 4x - x + 3y - 2z + 5z + 7 - 5$

(c) $4p + 2q - 5r - 1 - 1\ (7p - q + 3r + 3)$
$= 4p + 2q - 5r - 1 - 7p + q - 3r - 3$
$= 4p - 7p + 2q + q - 5r - 3r - 1 - 3$
$= -3p + 3q - 8r - 4$

(d) $6(2 + 3n + 5n - 4m\ (n + 5) - [2(3m - 5n) + 5mn]$
$= 12 + 18n + 30m - 4mn - 20m - (6m - 10n + 5mn)$
$= 12 + 18n + 30m - 4mn - 20m - 6m + 10n - 5mn)$
$= 12 + 18n + 10n + 30m - 20m - 6m - 4mn - 5mn$
$= 12 + 28n + 4m - 9mn$

25. (a) Let the second number be n.
Then the first number is $n - 2$.
Then the third number is $n + 2$.
Lastly, the fourth number is $(n + 2) + 2 = n + 4$.

(b) Sum of the four numbers
$= n - 2 + n + n + 2 + n + 4$
$= n + n + n + n - 2 + 2 + 4$
$= 4n + 4$

Advanced

26. (a) Observe that the pattern is alternate cube and square of the numbers in the sequence.
Add the term number to the previous term.
$\therefore$ The missing terms are $(23 + 7)^3 = 30^3$ and $(30 + 8)^2 = 38^2$.

(b) Observe the pattern as taking the cube of prime numbers.
$\therefore$ The missing terms are 5^3, 7^3 and 17^3.

(c) Observe that the pattern is taking the square of the prime numbers.
$\therefore$ The missing terms are 19^2, 17^2 and 11^2.

27. (a) $a(5b - 3) - b(4a - 1) + a(1 - 2b)$
$= 5ab - 3a - 4ab + b + a - 2ab$
$= 5ab - 4ab - 2ab - 3a + a + b$
$= -ab - 2a + b$

(b) $3x - \{2x - 4(x - 3y) - [(3x - 4y) - (y - 2x)]\}$
$= 3x - \{2x - 4(x - 3y) - [3x - 4y - y + 2x]\}$
$= 3x - \{2x - 4(x - 3y) - [3x + 2x - 4y - y]\}$
$= 3x - \{2x - 4x + 12y - [5x - 5y]\}$
$= 3x - \{- 2x + 12y - 5x + 5y]\}$
$= 3x - \{- 2x - 5x + 12y + 5y]\}$
$= 3x - \{- 7x + 17y\}$
$= 3x + 7x - 17y$
$= 10x - 17y$

New Trend

28. (a) $2(3x - 5) - 3(7 - 4x)$
$= 6x - 10 - 21 + 12x$
$= 6x + 12x - 10 - 21$
$= 18x - 31$

(b) $4(2x + 3y) - 7(x - 2y) = 8x + 12y - 7x + 14y$
$= x + 26y$

Chapter 6 Linear Equations

Basic

1. **(a)** $5x + 2 = 7$

$5x + 2 - 2 = 7 - 2$

$5x = 5$

$\frac{5x}{5} = \frac{5}{5}$

$x = 1$

(b) $2x - 7 = 3$

$2x - 7 + 7 = 3 + 7$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

(c) $15 - 2x = 9$

$15 - 2x + 2x = 9 + 2x$

$15 = 9 + 2x$

$9 + 2x - 9 = 15 - 9$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

(d) $17 + 3x = -3$

$17 + 3x - 17 = -3 - 17$

$3x = -20$

$\frac{3x}{3} = \frac{-20}{3}$

$x = -6\frac{2}{3}$

(e) $-4x + 7 = -15$

$-4x + 7 - 7 = -15 - 7$

$-4x = -22$

$\frac{-4x}{-4} = \frac{-22}{-4}$

$x = 5\frac{1}{2}$

(f) $2x - 3 = x + 5$

$2x - 3 + 3 = x + 5 + 3$

$2x = x + 8$

$2x - x = x + 8 - x$

$x = 8$

(g) $9x + 4 = 3x - 9$

$9x + 4 - 4 = 3x - 9 - 4$

$9x = 3x - 13$

$9x - 3x = 3x - 13 - 3x$

$6x = -13$

$\frac{6x}{6} = \frac{-13}{6}$

$x = -2\frac{1}{6}$

(h) $7x - 14 = 18 - 4x$

$7x - 14 + 14 = 18 - 4x + 14$

$7x = 32 - 4x$

$7x + 4x = 32 - 4x + 4x$

$11x = 32$

$\frac{11x}{11} = \frac{32}{11}$

$x = 2\frac{10}{11}$

2. **(a)** $3(x - 4) = 7$

$3x - 12 = 7$

$3x - 12 + 12 = 7 + 12$

$3x = 19$

$\frac{3x}{3} = \frac{19}{3}$

$x = 6\frac{1}{3}$

(b) $5(2x + 3) = 35$

$10x + 15 = 35$

$10x + 15 - 15 = 35 - 15$

$10x = 20$

$\frac{10x}{10} = \frac{20}{10}$

$x = 2$

(c) $4(3 - x) = -15$

$12 - 4x = -15$

$12 - 4x - 12 = -15 - 12$

$-4x = -27$

$\frac{-4x}{-4} = \frac{-27}{-4}$

$x = 6\frac{3}{4}$

(d) $2(7 - 2x) = 11$

$14 - 4x = 11$

$14 - 4x - 14 = 11 - 14$

$-4x = -3$

$\frac{-4x}{-4} = \frac{-3}{-4}$

$x = \frac{3}{4}$

(e) $2(x-5) = 5x + 7$

$2x - 10 = 5x + 7$

$2x - 10 - 7 = 5x + 7 - 7$

$2x - 17 = 5x$

$2x - 17 - 2x = 5x - 2x$

$-17 = 3x$

$3x = -17$

$\frac{3x}{3} = \frac{-17}{3}$

$x = -5\frac{2}{3}$

(f) $6 - 4x = 5(x - 6)$

$6 - 4x = 5x - 30$

$6 - 4x + 4x = 5x - 30 + 4x$

$6 = 9x - 30$

$6 + 30 = 9x - 30 + 30$

$36 = 9x$

$9x = 36$

$\frac{9x}{9} = \frac{36}{9}$

$x = 4$

(g) $2x - 3(5 - x) = 35$

$2x - 15 + 3x = 35$

$2x + 3x - 15 = 35$

$5x - 15 = 35$

$5x - 15 + 15 = 35 + 15$

$5x = 50$

$\frac{5x}{5} = \frac{50}{5}$

$x = 10$

(h) $7(x + 4) = 2(x - 4)$

$7x + 28 = 2x - 8$

$7x + 28 - 2x = 2x - 8 - 2x$

$5x + 28 = -8$

$5x + 28 - 28 = -8 - 28$

$5x = -36$

$\frac{5x}{5} = \frac{-36}{5}$

$x = -7\frac{1}{5}$

(i) $2(5 - 2x) = 4(2 - 3x)$

$10 - 4x = 8 - 12x$

$10 - 4x + 12x = 8 - 12x + 12x$

$8x + 10 = 8$

$8x + 10 - 10 = 8 - 10$

$8x = -2$

$\frac{8x}{8} = \frac{-2}{8}$

$x = -\frac{1}{4}$

(j) $(5x + 3) - (4x - 9) = 0$

$5x + 3 - 4x + 9 = 0$

$5x - 4x + 3 + 9 = 0$

$x + 12 = 0$

$x + 12 - 12 = 0 - 12$

$x = -12$

(k) $7(3 - 4x) - 5(2x + 8) = 0$

$21 - 28x - 10x - 40 = 0$

$21 - 40 - 28x - 10x = 0$

$-19 - 38x = 0$

$-19 - 38x + 19 = 0 + 19$

$-38x = 19$

$\frac{-38x}{-38} = \frac{19}{-38}$

$x = -\frac{1}{2}$

(l) $5(2x - 3) - 3(x - 2) = 0$

$10x - 15 - 3x + 6 = 0$

$10x - 3x - 15 + 6 = 0$

$7x - 9 = 0$

$7x - 9 + 9 = 0 + 9$

$7x = 9$

$\frac{7x}{7} = \frac{9}{7}$

$x = 1\frac{2}{7}$

3. (a) $\frac{3}{4}x = 15$

$\frac{3}{4}x \times 4 = 15 \times 4$

$3x = 60$

$\frac{3x}{3} = \frac{60}{3}$

$x = 20$

(b) $\frac{2}{5}x - 1 = 4$

$\frac{2}{5}x - 1 + 1 = 4 + 1$

$\frac{2}{5}x = 5$

$\frac{2}{5}x \times 5 = 5 \times 5$

$2x = 25$

$\frac{2x}{2} = \frac{25}{2}$

$x = 12\frac{1}{2}$

(c) $5 - \frac{3}{4}x = -1$

$5 - \frac{3}{4}x + \frac{3}{4}x = -1 + \frac{3}{4}x$

$5 = -1 + \frac{3}{4}x$

$5 + 1 = -1 + \frac{3}{4}x + 1$

$\frac{3}{4}x = 6$

$\frac{3}{4}x \times 4 = 6 \times 4$

$3x = 24$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

(d) $3 + \frac{4}{7}x = 1\frac{1}{3}$

$3 + \frac{4}{7}x - 3 = 1\frac{1}{3} - 3$

$\frac{4}{7}x = -1\frac{2}{3}$

$\frac{4}{7}x \times 7 = -1\frac{2}{3} \times 7$

$4x = -11\frac{2}{3}$

$\frac{4x}{4} = \frac{-11\frac{2}{3}}{4}$

$x = -2\frac{11}{12}$

(e) $2x = 0.4x + 12.8$

$2x - 0.4x = 0.4x + 12.8 - 0.4x$

$1.6x = 12.8$

$\frac{1.6x}{1.6} = \frac{12.8}{1.6}$

$x = 8$

(f) $0.3x + 1.2 = 0.25 - 0.2x$

$0.3x + 1.2 + 0.2x = 0.25 - 0.2x + 0.2x$

$0.5x + 1.2 = 0.25$

$0.5x + 1.2 - 1.2 = 0.25 - 1.2$

$0.5x = -0.95$

$\frac{0.5x}{0.5} = \frac{-0.95}{0.5}$

$x = -1.9$

(g) $\frac{2}{3}x + 15 = 4x$

$\frac{2}{3}x + 15 - \frac{2}{3}x = 4x - \frac{2}{3}x$

$15 = 3\frac{1}{3}x$

$3\frac{1}{3}x = 15$

$\frac{3\frac{1}{3}x}{3\frac{1}{3}} = \frac{15}{3\frac{1}{3}}$

$x = 4\frac{1}{2}$

(h) $1.3x - 3.6 = \frac{4}{5}x + 2$

$1.3x - 3.6 - \frac{4}{5}x = \frac{4}{5}x + 2 - \frac{4}{5}x$

$0.5x - 3.6 = 2$

$0.5x - 3.6 + 3.6 = 2 + 3.6$

$0.5x = 5.6$

$\frac{0.5x}{0.5} = \frac{5.6}{0.5}$

$x = 11.2$

(i) $1.5 - \frac{7}{8}x = 2.6x + \frac{1}{5}$

$1.5 - \frac{7}{8}x + \frac{7}{8}x = 2.6x + \frac{1}{5} + \frac{7}{8}x$

$1.5 = 3.475x + \frac{1}{5}$

$1.5 - \frac{1}{5} = 3.475x + \frac{1}{5} - \frac{1}{5}$

$3.475x = 1.3$

$\frac{3.475x}{3.475} = \frac{1.3}{3.475}$

$x = \frac{52}{139}$

4. **(a)** $\frac{2x - 3}{5} = 7$

$\frac{2x - 3}{5} \times 5 = 7 \times 5$

$2x - 3 = 35$

$2x - 3 + 3 = 35 + 3$

$2x = 38$

$\frac{2x}{2} = \frac{38}{2}$

$x = 19$

(b) $\frac{3x-4}{5} - 7 = 0$

$$\frac{3x-4}{5} = 7$$

$$\frac{3x-4}{5} \times 5 = 7 \times 5$$

$$3x - 4 = 35$$

$$3x - 4 + 4 = 35 + 4$$

$$3x = 39$$

$$\frac{3x}{3} = \frac{39}{3}$$

$$x = 13$$

(c) $\frac{x+1}{3} = \frac{3x}{5}$

$$15 \times \frac{x+1}{3} = 15 \times \frac{3x}{5}$$

$$5(x+1) = 3(3x)$$

$$5x + 5 = 9x$$

$$5x + 5 - 5x = 9x - 5x$$

$$5 = 4x$$

$$4x = 5$$

$$\frac{4x}{4} = \frac{5}{4}$$

$$x = 1\frac{1}{4}$$

(d) $\frac{2x-1}{3} = 1 - x$

$$\frac{2x-1}{3} \times 3 = (1-x) \times 3$$

$$2x - 1 = 3(1-x)$$

$$2x - 1 = 3 - 3x$$

$$2x - 1 + 3x = 3 - 3x + 3x$$

$$5x - 1 = 3$$

$$5x - 1 + 1 = 3 + 1$$

$$5x = 4$$

$$\frac{5x}{5} = \frac{4}{5}$$

$$x = 0.8$$

(e) $\frac{2}{3}(5x-7) = \frac{4}{5}$

$$15 \times \frac{2}{3}(5x-7) = 15 \times \frac{4}{5}$$

$$10(5x-7) = 12$$

$$50x - 70 = 12$$

$$50x - 70 + 70 = 12 + 70$$

$$50x = 82$$

$$\frac{50x}{50} = \frac{82}{50}$$

$$x = 1\frac{16}{25}$$

(f) $\frac{2}{3}(6x+5) = 7(x-4.5)$

$$4x + 3\frac{1}{3} = 7x - 31.5$$

$$4x + 3\frac{1}{3} - 4x = 7x - 31.5 - 4x$$

$$3\frac{1}{3} = 3x - 31.5$$

$$3\frac{1}{3} + 31.5 = 3x - 31.5 + 31.5$$

$$\frac{209}{6} = 3x$$

$$3x = \frac{209}{6}$$

$$\frac{3x}{3} = \frac{\frac{209}{6}}{3}$$

$$x = 11\frac{11}{18}$$

(g) $\frac{1}{4}(3x+5) = \frac{1}{3}(5x-4)$

$$12 \times \frac{1}{4}(3x+5) = 12 \times \frac{1}{3}(5x-4)$$

$$3(3x+5) = 4(5x-4)$$

$$9x + 15 = 20x - 16$$

$$9x + 15 - 15 = 20x - 16 - 15$$

$$9x = 20x - 31$$

$$9x - 20x = 20x - 31 - 20x$$

$$-11x = -31$$

$$11x = 31$$

$$\frac{11x}{11} = \frac{31}{11}$$

$$x = 2\frac{9}{11}$$

(h) $\frac{1}{5}(4-3x) = \frac{1}{7}(3x-4)$

$$35 \times \frac{1}{5}(4-3x) = 35 \times \frac{1}{7}(3x-4)$$

$$7(4-3x) = 5(3x-4)$$

$$28 - 21x = 15x - 20$$

$$28 - 21x + 21x = 15x - 20 + 21x$$

$$28 = 36x - 20$$

$$28 + 20 = 36x - 20 + 20$$

$$48 = 36x$$

$$36x = 48$$

$$\frac{36x}{36} = \frac{48}{36}$$

$$x = 1\frac{1}{3}$$

(i) $\dfrac{4x-3}{5} = \dfrac{2x-7}{8}$

$8(4x-3) = 5(2x-7)$

$32x-24 = 10x-35$

$32x-10x = -35+24$

$22x = -11$

$\dfrac{22x}{22} = \dfrac{-11}{22}$

$x = -\dfrac{1}{2}$

5. **(a)** $y = a(4a-5)$

When $a = 3$, $y = 3(4\times 3-5)$

$= 3(7) = 21$

(b) $y = (x+p)(3x-p-4)$

When $x = 3$, $p = 4$,

$y = (3+4)(3\times 3-4-4)$

$= (7)(1) = 7$

(c) $y = \dfrac{2x-1}{3}$

When $x = 5$, $y = \dfrac{2(5)-1}{3} = \dfrac{9}{3} = 3$

(d) $y = \dfrac{2r+5}{7r-9}$

When $r = 6$,

$y = \dfrac{2(6)+5}{7(6)-9}$

$= \dfrac{17}{33}$

6. $xy - 3y^2 = 15$

When $y = 2$,

$x(2) - 3(2)^2 = 15$

$2x - 12 = 15$

$2x = 15+12$

$2x = 27$

$x = 13\dfrac{1}{2}$

7. $y = \dfrac{2}{3}(24-x) + 5xy$

When $x = -3\dfrac{1}{3}$,

$y = \dfrac{2}{3}\left[24 - \left(-3\dfrac{1}{3}\right)\right] + 5\left(-3\dfrac{1}{3}\right)y$

$y = \dfrac{2}{3}\left(27\dfrac{1}{3}\right) - 16\dfrac{2}{3}y$

$y = 18\dfrac{2}{9} - 16\dfrac{2}{3}y$

$y + 16\dfrac{2}{3}y = 18\dfrac{2}{9}$

$17\dfrac{2}{3}y = 18\dfrac{2}{9}$

$y = 1\dfrac{5}{159}$

8. $p - 5q = 4qr$

When $q = 4$, $r = -1$,

$p - 5(4) = 4(4)(-1)$

$p - 20 = -16$

$p = -16+20 = 4$

9. **(a)** $D = a^2 - b^2$

(b) The three consecutive numbers are d, $d+2$ and $d+4$.

$S = d + (d+2) + (d+4) = 3d+6 = 3(d+2)$

(c) Perimeter of square $= m+m+m+m = 4m$

Perimeter of rectangle $= 2(n+s)$

Perimeter of figure,

$P = 4m + 2(n+s)$

10. **(a)** Let the smallest odd number be n.

The next odd number is $n+2$.

The largest odd number is $(n+2)+2 = n+4$.

$\therefore S = n+n+2+n+4 = 3n+6$

$3n+6 = 243$

$3n = 243-6 = 237$

$n = 79$

$\therefore$ The largest odd number is $79+4 = 83$.

(b) Let the smallest even number be n.

The next even number is $n+2$.

The next even number is $(n+2)+2 = n+4$.

The next even number is $(n+4)+2 = n+6$.

The largest even number is $(n+6)+2 = n+8$.

$\therefore S = n+n+2+n+4+n+6+n+8$

$= 5n+20$

$5n+20 = 220$

$5n = 220-20 = 200$

$n = 40$

$\therefore$ The smallest of the five numbers is 40.

(c) Let the smaller odd number be n.

The next odd number is $n+2$.

$3(n+2) - n = 56$

$3n+6-n = 56$

$2n = 56-6$

$2n = 50$

$n = 25$

$\therefore$ The two numbers are 25 and 27.

(d) Let the smaller even number be n.

The next even number is $n+2$.

$n+2+3n = 42$

$4n = 40$

$n = 10$

$\therefore$ The two numbers are 10 and 12.

11. (a) Let the age of Ali be x years old.
Then Rizwan is $2x$ years old.
Hussain is $(2x - 7)$ years old.

$$x + 2x + (2x - 7) = 38$$
$$5x = 38 + 7$$
$$5x = 45$$
$$x = 9$$

Ali is 9 years old.
Rizwan is $2 \times 9 = 18$ years old.
Hussain is $(2 \times 9 - 7) = 11$ years old.

(b) Let the number of years ago in which Kiran's father is three times as old as her be n.

$$50 - n = 3(24 - n)$$
$$50 - n = 72 - 3n$$
$$2n = 72 - 50$$
$$2n = 22$$
$$n = 11$$

$\therefore$ Kiran's father was three times as old as Kiran 11 years ago.

(c) Let the age of Farhan be x years old.
Then Farhan's brother's age is $3x$ years old.
In 12 years' time,
Farhan will be $(x + 12)$ years old and his brother will be $(3x + 12)$ years old.

$$(x + 12) + (3x + 12) = 10x$$
$$4x + 24 = 10x$$
$$6x = 24$$
$$x = 4$$

$\therefore$ Farhan's present age is 4 years old and his brother is 12 years old.

12. (a) Let the first number be x.
Then the second number is $120 - x$.

$$120 - x = 4x$$
$$5x = 120$$
$$x = 24$$

$\therefore$ The smaller number is 24.

(b) Let the number be x.

$$12 - \frac{x}{4} = \frac{1}{6}x$$
$$12 = \frac{1}{6}x + \frac{x}{4}$$
$$12 = \frac{5}{12}x$$
$$144 = 5x$$
$$x = 28\frac{4}{5}$$

$\therefore$ The number is $28\frac{4}{5}$.

13. (a) The cost of 12 pears is equal to the cost of 36 apples.
A pear costs 3 times an apple.
Let the cost of an apple be PKR x.
Then the cost of a pear is PKR $3x$.
The amount of money Maaz has is PKR $36x$.
Cost of 1 apple and 1 pear
= PKR $3x$ + PKR x
= PKR $4x$
No. of each fruit Maaz can buy

$$= \frac{36x}{4x}$$
$$= 9$$

(b) Amount of money spent on pencils

$$= 15 \times \frac{2x}{100} = \text{PKR } \frac{3x}{10}$$

Amount of money spent on pens

$$= 24 \times \frac{4y}{100} = \text{PKR } \frac{24y}{25}$$

Total amount spent on pencils and pens

$$= \frac{3x}{10} + \frac{24y}{25}$$
$$= \text{PKR } \frac{15x + 48y}{50}$$

Intermediate

14. (a)
$$5(3x - 2) - 7(x - 1) = 12$$
$$15x - 10 - 7x + 7 = 12$$
$$15x - 7x - 10 + 7 = 12$$
$$8x - 3 = 12$$
$$8x = 12 + 3$$
$$8x = 15$$
$$x = \frac{15}{8}$$
$$= 1\frac{7}{8}$$

(b)
$$4(3 - x) + 3(4x + 5) = -45$$
$$12 - 4x + 12x + 15 = -45$$
$$-4x + 12x + 12 + 15 = -45$$
$$8x + 27 = -45$$
$$8x = -45 - 27$$
$$8x = -72$$
$$x = -9$$

(c) $0.3(4x - 1) = 0.8 + x$

$1.2x - 0.3 = 0.8 + x$

$1.2x - x = 0.8 + 0.3$

$0.2x = 1.1$

$\frac{0.2x}{0.2} = \frac{1.1}{0.2}$

$x = 5.5$

(d) $3(5x + 2) - 7(3 - x) = (19 + 5x) + (20 - x)$

$15x + 6 - 21 + 7x = 19 + 20 + 5x - x$

$15x + 7x - 15 = 39 + 4x$

$22x - 15 = 39 + 4x$

$22x - 4x = 39 + 15$

$18x = 54$

$x = 3$

(e) $2x - [3 + 5(x - 5)] = 10$

$2x - [3 + 5x - 25] = 10$

$2x - [5x - 22] = 10$

$2x - 5x + 22 = 10$

$-3x = 10 - 22$

$-3x = -12$

$x = 4$

(f) $3x - [3 - 2(3x - 7)] = 37$

$3x - [3 - 6x + 14] = 37$

$3x - [17 - 6x] = 37$

$3x - 17 + 6x = 37$

$3x + 6x = 37 + 17$

$9x = 54$

$x = 6$

15. (a) $\frac{2(x - 1)}{3} + \frac{3x}{4} = 0$

$12 \times \frac{2(x - 1)}{3} + \frac{3x}{4} = 12 \times 0$

$8(x - 1) + 9x = 0$

$8x - 8 + 9x = 0$

$8x + 9x = 8$

$17x = 8$

$x = \frac{8}{17}$

(b) $\frac{6x + 1}{7} - \frac{2x - 7}{3} = 4$

$21 \times \left(\frac{6x + 1}{7} - \frac{2x - 7}{3}\right) = 21 \times 4$

$3(6x + 1) - 7(2x - 7) = 84$

$18x + 3 - 14x + 49 = 84$

$4x + 52 = 84$

$4x = 84 - 52$

$4x = 32$

$x = 8$

(c) $2x - \frac{x}{4} + \frac{3x}{5} = 14 + \frac{7x}{3}$

$2x - \frac{x}{4} + \frac{3x}{5} - \frac{7x}{3} = 14$

$\frac{x}{60} = 14$

$60 \times \frac{x}{60} = 60 \times 14$

$x = 840$

(d) $5x - 1\frac{3}{4} = 6 + 1\frac{2}{3}x - \frac{5}{6}$

$5x - 1\frac{3}{4}x = 5\frac{1}{6} + 1\frac{2}{3}x$

$5x - 1\frac{2}{3}x = 5\frac{1}{6} + 1\frac{3}{4}$

$3\frac{1}{3}x = 6\frac{11}{12}$

$x = 2\frac{3}{40}$

(e) $\frac{x}{4} = \frac{x + 12}{10} + 0.6$

$\frac{x}{4} = \frac{x}{10} + \frac{12}{10} + 0.6$

$\frac{x}{4} - \frac{x}{10} = 1.2 + 0.6$

$\frac{3x}{20} = 1.8$

$x = 12$

(f) $\frac{3x - 4}{6} - \frac{2x + 3}{8} = \frac{2x - 7}{24}$

$24 \times \left(\frac{3x - 4}{6} - \frac{2x + 3}{8}\right) = 24 \times \frac{2x - 7}{24}$

$4(3x - 4) - 3(2x + 3) = 2x - 7$

$12x - 16 - 6x - 9 = 2x - 7$

$6x - 25 = 2x - 7$

$6x - 2x = -7 + 25$

$4x = 18$

$x = 4\frac{1}{2}$

(g) $\frac{5x - 1}{8} - \frac{5 - 7x}{2} = \frac{3(6 - x)}{6}$

$24 \times \left(\frac{5x - 1}{8} - \frac{5 - 7x}{2}\right) = 24 \times \frac{3(6 - x)}{6}$

$3(5x - 1) - 12(5 - 7x) = 12(6 - x)$

$15x - 3 - 60 + 84x = 72 - 12x$

$99x - 64 = 72 - 12x$

$99x + 12x = 72 + 63$

$111x = 135$

$x = 1\frac{8}{37}$

(h) $\frac{5x+2}{7} = \frac{x-3}{5} + x + 1.5$

$35 \times \frac{5x+2}{7} = 35 \times \left(\frac{x-3}{5} + x + 1.5\right)$

$5(5x+2) = 7(x-3) + 35x + 52.5$

$25x + 10 = 7x - 21 + 35x + 52.5$

$25x + 10 = 42x + 31.5x$

$25x - 42x = 31.5 - 10$

$-17x = 21.5$

$17x = -21.5$

$x = -1\frac{9}{34}$

(i) $\frac{x}{3} - \frac{7(x-2)}{9} = 4 - \frac{2x-5}{6}$

$18 \times \left(\frac{x}{3} - \frac{7(x-2)}{9}\right) = 18 \times \left(4 - \frac{2x-5}{6}\right)$

$6(x) - 14(x-2) = 72 - 3(2x-5)$

$6x - 14x + 28 = 72 - 6x + 15$

$-8x + 28 = 87 - 6x$

$-8x + 6x = 87 - 28$

$-2x = 59$

$x = -29.5$

(j) $0.5x + 2 = \frac{1}{4} + \frac{x-1}{2} + \frac{x}{4} - \frac{1}{6}$

$0.5x + 2 = \frac{1}{4} + \frac{x}{2} - \frac{1}{2} + \frac{x}{4} - \frac{1}{6}$

$0.5x - \frac{x}{2} - \frac{x}{4} = \frac{1}{4} - \frac{1}{2} - \frac{1}{6} - 2$

$-\frac{x}{4} = -2\frac{5}{12}$

$x = 9\frac{2}{3}$

(k) $4x + 1 - \frac{1}{2}(3x-2) - \frac{1}{3}(4x-1) = 0$

$6 \times \left(4x + 1 - \frac{1}{2}(3x-1) - \frac{1}{3}(4x-1)\right) = 6 \times 0$

$24x + 6 - 3(3x-2) - 2(4x-1) = 0$

$24x + 6 - 9x + 6 - 8x + 2 = 0$

$24x - 9x - 8x + 6 + 6 + 2 = 0$

$7x + 14 = 0$

$7x = -14$

$x = -2$

(l) $\frac{1}{2}\left(2x - \frac{1}{2}\right) = \frac{1}{3}\left(3x - \frac{1}{4}\right) + \frac{1}{4}(4x-3)$

$x - \frac{1}{4} = x - \frac{1}{12} + x - \frac{3}{4}$

$x - x - x = -\frac{1}{12} - \frac{3}{4} + \frac{1}{4}$

$-x = -\frac{7}{12}$

$x = \frac{7}{12}$

16. (a) $\frac{3}{x} + \frac{4}{x} = 5$

$\frac{7}{x} = 5$

$x \times \frac{7}{x} = x \times 5$

$7 = 5x$

$x = \frac{7}{5} = 1\frac{2}{5}$

(b) $\frac{5}{2x} - \frac{7}{5x} = \frac{2}{3}$

$10x \times \left(\frac{5}{2x} - \frac{7}{5x}\right) = 10x \times \frac{2}{3}$

$25 - 14 = 6\frac{2}{3}x$

$11 = 6\frac{2}{3}x$

$x = 1\frac{13}{20}$

(c) $\frac{7}{2x} + \frac{5}{3x} = 1\frac{5}{6}$

$6x \times \left(\frac{7}{2x} + \frac{5}{3x}\right) = 6x \times 1\frac{5}{6}$

$21 + 10 = 11x$

$31 = 11x$

$x = 2\frac{9}{11}$

(d) $\frac{5}{x+2} - \frac{4}{2x+4} = 6$

$\frac{5}{x+2} - \frac{4}{2(x+2)} = 6$

$\frac{10}{2(x+2)} - \frac{4}{2(x+2)} = 6$

$\frac{6}{2(x+2)} = 6$

$12(x+2) = 6$

$12x + 24 = 6$

$12x = 6 - 24$

$12x = -18$

$x = -1\frac{1}{2}$

(e) $1 - \frac{x+1}{3x+5} = \frac{1}{2}$

$\frac{x+1}{3x+5} = 1 - \frac{1}{2}$

$\frac{x+1}{3x+5} = \frac{1}{2}$

$2(x+1) = 3x + 5$

$2x + 2 = 3x + 5$

$3x - 2x = 2 - 5$

$x = -3$

17. $5(2x - 3) - 3(x - 2) = 0$

$10x - 15 - 3x + 6 = 0$

$10x - 3x - 15 + 6 = 0$

$7x - 9 = 0$

$7x - 9 + 11 = 0 + 11$

$7x + 2 = 11$

18. When $x = 4$,

LHS

$= -2 - \frac{2 \times 4}{5} + \frac{3 \times 4}{2}$

$= -2 - \frac{8}{5} + \frac{12}{2} = 2\frac{2}{5} \neq 4\frac{3}{5}$ (RHS)

$\therefore$ No, $x = 4$ is not a solution of the equation.

19. When $y = 2$, $p = 5$ and $q = 6$,

$x - 2 = \frac{x(2)}{5 - 6}$

$x - 2 = \frac{2x}{-1}$

$x - 2 = -2x$

$x + 2x = 2$

$3x = 2$

$x = \frac{2}{3}$

20. When $y = 8$ and $z = 2$,

$\frac{x - 1}{8 + 3} - \frac{x}{8} = \frac{1}{2}$

$\frac{x - 1}{11} - \frac{x}{8} = \frac{1}{2}$

$\frac{x}{11} - \frac{1}{11} - \frac{x}{8} = \frac{1}{2}$

$\frac{x}{11} - \frac{x}{8} = \frac{1}{2} + \frac{1}{11}$

$-\frac{3}{88}x = \frac{13}{22}$

$x = -17\frac{1}{3}$

21. When $a = 3\frac{1}{2}$, $h = 10$ and $k = 15$,

$\frac{1}{x} = \left(3\frac{1}{2} - 2\right)\left(\frac{1}{10} + \frac{1}{15}\right)$

$= \left(1\frac{1}{2}\right)\left(\frac{1}{6}\right)$

$= \frac{1}{4}$

$\therefore x = 4$

22. When $y = 6$ and $z = -\frac{1}{2}$,

$\frac{3x + 2(6) - 5\left(-\frac{1}{2}\right)}{6 - 4\left(-\frac{1}{2}\right)} = \frac{x}{3(6)}$

$\frac{3x + 12 + 2\frac{1}{2}}{8} = \frac{x}{18}$

$\frac{3x + 14\frac{1}{2}}{8} = \frac{x}{18}$

$8 \times \frac{3x + 14\frac{1}{2}}{8} = 8 \times \frac{x}{18}$

$3x + 14\frac{1}{2} = \frac{4x}{9}$

$3x - \frac{4x}{9} = -14\frac{1}{2}$

$2\frac{5}{9}x = -14\frac{1}{2}$

$x = -5\frac{31}{46}$

23. When $p = 3$, $q = -2$,

$\frac{5(3) - 3(-2)}{r} = \frac{3(-2) - 5(3)}{3 + (-2)}$

$\frac{15 + 6}{r} = \frac{-6 - 15}{1}$

$\frac{21}{r} = \frac{-21}{1}$

$1(21) = r(-21)$

$21 = -21r$

$r = -1$

24. $A = P + \frac{PRT}{100}$

(a) When $P = 5000$, $R = 5$ and $T = 3$,

$A = 5000 + \frac{(5000)(5)(3)}{100}$

$= 5750$

(b) When $A = 6500$, $R = 5$ and $T = 1\frac{2}{3}$,

$6500 = P + \frac{P(5)\left(1\frac{2}{3}\right)}{100}$

$6500 = P + \frac{1}{12}P$

$6500 = 1\frac{1}{12}P$

$1\frac{1}{12}P = 6500$

$P = 6500 \div 1\frac{1}{12} = 6000$

25. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

(a) When $u = 5$ and $v = 7$,

$\frac{1}{f} = \frac{1}{5} + \frac{1}{7}$

$\frac{1}{f} = \frac{12}{35}$

$12f = 35$

$\therefore f = \frac{35}{12} = 2\frac{11}{12}$

(b) When $f = 4$ and $v = 5$,

$\frac{1}{4} = \frac{1}{u} + \frac{1}{5}$

$\frac{1}{u} = \frac{1}{4} - \frac{1}{5}$

$\frac{1}{u} = \frac{1}{20}$

$\therefore u = 20$

26. (a) (i) Let the first number be x.

Then the second number is mx.

Then the third number is $mx - n$.

Sum of the three numbers $S = x + mx + mx - n$

$= x + 2mx - n$

(ii) When $S = 109$, $m = 4$, $n = 8$,

$109 = x + 2(4)x - 8$

$109 = x + 8x - 8$

$9x = 109 + 8$

$9x = 117$

$\therefore x = 13$

The three numbers are 13, $4(13) = 52$ and $52 - 8 = 44$.

(b) (i) The cost of the pair of shoes is PKR C.

Amount of money Nadia has after buying the pair of shoes = PKR $(p - C)$

Amount of money Seema has after buying the pair of shoes = PKR $(q - C)$

$p - C = 2(q - C)$

$p - C = 2q - 2C$

$2C - C = 2q - p$

$C = 2q - p$

(ii) When $p = 42$, $q = 30$,

cost of the pair of shoes $= 2 \times 30 - 42$

= PKR 18

27. (i) Let the number Faiza is thinking of be x.

$2x + 14 = 4x - 8$

(ii) $2x + 14 = 4x - 8$

$14 + 8 = 4x - 2x$

$22 = 2x$

$x = 11$

(iii) The result is $2x + 14 = 2(11) + 14 = 36$.

28. Let the denominator of the fraction be x.

Then the numerator is $x - 1$.

$\frac{x - 1 + 1}{x + 2} = \frac{3}{4}$

$\frac{x}{x + 2} = \frac{3}{4}$

$4x = 3(x + 2)$

$4x = 3x + 6$

$4x - 3x = 6$

$x = 6$

Then the numerator is $6 - 1 = 5$.

The original fraction is $\frac{5}{6}$.

29. (i) The woman's present age is $8x$ years old

(ii) Maaz's age two years ago was $(x - 2)$ years old.

(iii) The woman's age two years ago was

$= (8x - 2)$ years old

$8x - 2 = 15(x - 2)$

$8x - 2 = 15x - 30$

$8x - 15x = -30 + 2$

$-7x = -28$

$7x = 28$

$x = 4$

(iv) The woman's present age $= 8 \times 4 = 32$ years old.

The woman's age in 5 years' time

$= 32 + 5$

$= 37$ years old

30. (i) Amount of time spent cycling $= \frac{x}{9}$ hours

(ii) Amount of time spent taking the train

$= \frac{28}{60} - \frac{x}{9} - \frac{3}{60} - \frac{\frac{1}{2}}{6}$

$= \frac{7}{15} - \frac{x}{9} - \frac{3}{60} - \frac{1}{12}$

$= \left(\frac{1}{3} - \frac{x}{9}\right)$ hours

Distance travelled by Ahsan on the MRT train

$= 60\left(\frac{1}{3} - \frac{x}{9}\right)$

$= \left(20 - 6\frac{2}{3}x\right)$ km

(iii) $x + 20 - 6\frac{2}{3}x + \frac{1}{2} = 12$

$$6\frac{2}{3}x - x = 20 + \frac{1}{2} - 12$$

$$5\frac{2}{3}x = 8\frac{1}{2}$$

$$x = 1\frac{1}{2}$$

31. Let the number of apples bought be x.
Then the number of oranges bought is $2x$.
Then the number of pears bought is $(x - 5)$.

(i) Amount spent on the fruits = PKR 77

$x(0.40) + 2x(0.30) + (x - 5)(0.80) = 77$

$0.4x + 0.6x + 0.8x - 4 = 77$

$1.8x - 4 = 77$

$1.8x = 77 + 4$

$1.8x = 81$

$x = 45$

(ii) Amount of money spent on buying the pears
$= (x - 5)(0.80)$
$= (45 - 5)(0.80)$
$= (40)(0.80)$
= PKR 32
He spent PKR 32 on buying the pears.

32. Let the number of ducks bought be x.
Then the number of chicken bought is $3x$.
The number of geese bought is $0.5x$.

Total cost = PKR 607.20

$x(7.5) + 3x(3.8) + 0.5x(12.8) = 607.2$

$7.5x + 11.4x + 6.4x = 607.2$

$25.3x = 607.2$

$x = 24$

The number of geese bought is $0.5 \times 24 = 12$.

33. (i) Amount of money the salesman earned in a week

$= 90 + \frac{12(580)}{100}$
$= 90 + 69.60$
= PKR 159.60

(ii) To find the number of articles sold, make n the subject.

$$A = 90 + \frac{12n}{100}$$

$$A - 90 = \frac{12n}{100}$$

$$12n = 100(A - 90)$$

$$n = \frac{100(A - 90)}{12}$$

$$= \frac{100(190.80 - 90)}{12}$$

$$= \frac{100(100.80)}{12}$$

$$= 840$$

(iii) $A = 80 + \frac{16n}{100}$

(iv) For the same amount of money earned before and after

$$90 + \frac{12n}{100} = 80 + \frac{16n}{100}$$

$$90 - 80 = \frac{16n}{100} - \frac{12n}{100}$$

$$\frac{n}{25} = 10$$

$$n = 250$$

The number of articles the salesman must sell in a week to earn the same amount of money before and after the adjustments is 250.

34. (i) Rizwan's brother's age is $0.5 \times 4x = 2x$ years old.
Sum of their present ages = $4x + 2x = 6x$ years old

(ii) In 8 years' time,
Rizwan is $(4x + 8)$ years old and his brother is $(2x + 8)$ years old.
Sum of their ages in 8 years' time
$= (4x + 8) + (2x + 8)$
$= 4x + 2x + 8 + 8$
$= (6x + 16)$ years old

35. Let the second number be x.
Then the first number is $(x + 5)$.
Then the third number is $0.5x$.
The fourth number is $3[(x + 5) + x] = 3(2x + 5)$.
The total of the four numbers is $56 \times 4 = 224$.

$(x + 5) + x + 0.5x + 3(2x + 5) = 224$

$x + 5 + x + 0.5x + 6x + 15 = 224$

$x + x + 0.5x + 6x + 5 + 15 = 224$

$8.5x + 20 = 224$

$8.5x = 224 - 20$

$8.5x = 204$

$x = 24$

The numbers are $24 + 5 = 29$, 24, $0.5(24) = 12$ and $3(2 \times 24 + 5) = 159$.

36. Let the first number be x.
Let the second number be $84 - x$.

$$\frac{1}{2}x - \frac{1}{3}(84 - x) = 2$$

$$\frac{1}{2}x + \frac{1}{3}x - 28 = 2$$

$$\frac{5}{6}x = 2 + 28$$

$$\frac{5}{6}x = 30$$

$$5x = 180$$

$$x = 36$$

The two numbers are 36 and 48.

37. In 1 hour, Ali can complete $\frac{1}{3}$ of the task.

In 1 hour, Farhan can complete $\frac{1}{2}$ of the task.

In 1 hour, when they work together, they can complete

$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ of the task

$\therefore$ It takes them $\frac{6}{5}$ hours = 1 hour and 12 minutes to complete the task.

New Trend

38. $\frac{2x-1}{3} - \frac{3x-4}{5} = \frac{4}{7}$

$15 \times \left(\frac{2x-1}{3} - \frac{3x-4}{5}\right) = 15 \times \frac{4}{7}$

$5(2x-1) - 3(3x-4) = 8\frac{4}{7}$

$10x - 5 - 9x + 12 = 8\frac{4}{7}$

$x + 7 = 8\frac{4}{7}$

$x = 8\frac{4}{7} - 7$

$x = 1\frac{4}{7}$

39. $\frac{3}{2x+5} = \frac{4}{1-3x}$

$3(1-3x) = 4(2x+5)$

$3 - 9x = 8x + 20$

$9x + 8x = 3 - 20$

$17x = -17$

$x = -1$

40. $5(2-3x) - (1+7x) = 5(3-6x)$

$10 - 15x - 1 - 7x = 15 - 30x$

$9 - 22x = 15 - 30x$

$-22x + 30x = 15 - 9$

$8x = 6$

$x = \frac{6}{8}$

$= \frac{3}{4}$

41. $\frac{3x+2}{4} = \frac{2x-1}{3}$

$12 \times \frac{3x+2}{4} = 12 \times \frac{2x-1}{3}$

$3(3x+2) = 4(2x-1)$

$9x + 6 = 8x - 4$

$9x - 8x = -4 - 6$

$x = -4 - 6$

$= -10$

Chapter 7 Rate and Ratio

Basic

1. **(a)** $14 : 35$

$14 \div 7 : 35 \div 7$

$2 : 5$

(b) $24 : 42$

$24 \div 6 : 42 \div 6$

$4 : 7$

(c) $36 : 132$

$36 \div 6 : 132 \div 6$

$6 : 22$

$6 \div 2 : 22 \div 2$

$3 : 11$

(d) $135 : 240$

$135 \div 5 : 240 \div 5$

$27 : 48$

$27 \div 3 : 48 \div 3$

$9 : 16$

(e) $144 : 128$

$144 \div 16 : 128 \div 16$

$9 : 8$

(f) $162 : 384$

$162 \div 6 : 384 \div 6$

$27 : 64$

(g) $192 : 75$

$192 \div 3 : 75 \div 3$

$64 : 25$

(h) $418 : 242$

$418 \div 2 : 242 \div 2$

$209 : 121$

$209 \div 11 : 121 \div 11$

$19 : 11$

2. **(a)** $\frac{9}{20} : \frac{3}{5} = \frac{9}{20} \times 20 : \frac{3}{5} \times 20$

$= 9 : 12$

$= 3 : 4$

(b) $\frac{7}{15} : \frac{14}{9} = \frac{7}{15} \times 9 : \frac{14}{9} \times 9$

$= \frac{21}{5} : 14$

$= \frac{21}{5} \times 5 : 14 \times 5$

$= 21 : 70$

$= 3 : 10$

(c) $\frac{15}{28} : \frac{18}{7} = \frac{15}{28} \times 28 : \frac{18}{7} \times 28$

$= 15 : 72$

$= 5 : 24$

(d) $\frac{25}{44} : \frac{50}{33} = \frac{25}{44} \times 11 : \frac{50}{33} \times 11$

$= \frac{25}{4} : \frac{50}{3}$

$= \frac{25}{4} \times 12 : \frac{50}{3} \times 12$

$= 75 : 200$

$= 3 : 8$

(e) $1\frac{25}{56} : \frac{18}{21} = \frac{81}{56} : \frac{18}{21}$

$= \frac{81}{56} \times 21 : \frac{18}{21} \times 21$

$= \frac{243}{8} : 18$

$= \frac{243}{8} \times 8 : 18 \times 8$

$= 243 : 144$

$= 27 : 16$

(f) $4\frac{1}{3} : 65 = \frac{13}{3} : 65$

$= \frac{13}{3} \times 3 : 65 \times 3$

$= 13 : 195$

$= 1 : 15$

(g) $8\frac{3}{4} : 3\frac{1}{8} = \frac{35}{4} : \frac{25}{8}$

$= \frac{35}{4} \times 8 : \frac{25}{8} \times 8$

$= 70 : 25$

$= 14 : 5$

(h) $2.4 : 1\frac{1}{5} = 2\frac{4}{10} : 1\frac{1}{5}$

$= \frac{12}{5} \times 5 : \frac{6}{5} \times 5$

$= 12 : 6$

$= 2 : 1$

3. **(a)** $0.09 : 0.21$

$0.09 \times 100 : 0.21 \times 100$

$9 : 21$

$3 : 7$

(b) $0.192 : 0.064$

$0.192 \times 1000 : 0.064 \times 1000$

$192 : 64$

$3 : 1$

(c) 0.25 : 1.5

$0.25 \times 100 : 1.5 \times 100$

25 : 150

1 : 6

(d) 0.63 : 9.45

$0.63 \times 100 : 9.45 \times 100$

63 : 945

1 : 15

(e) 0.84 : 1.12

$0.84 \times 100 : 1.12 \times 100$

84 : 112

21 : 28

3 : 4

(f) 1.26 : 0.315

$1.26 \times 1000 : 0.315 \times 1000$

1260 : 315

4 : 1

(g) 1.44 : 0.48

$1.44 \times 100 : 0.48 \times 100$

144 : 48

3 : 1

(h) 1.8 : 0.4

$1.8 \times 10 : 0.4 \times 10$

18 : 4

9 : 2

4. **(a)** 6 parts = PKR 336

$1 \text{ part} = \frac{336}{6} = \text{PKR } 56$

5 parts = 56×5 = PKR 280

∴ PKR 56 : PKR 280

(b) 14 parts = PKR 336

$1 \text{ part} = \frac{336}{14} = \text{PKR } 24$

3 parts = 24×3 = PKR 72

11 parts = 24×11 = PKR 264

∴ PKR 72 : PKR 264

(c) 16 parts = PKR 336

$1 \text{ part} = \frac{336}{16} = \text{PKR } 21$

3 parts = 21×3 = PKR 63

13 parts = 21×13 = PKR 273

∴ PKR 63 : PKR 273

(d) 8 parts = PKR 336

$1 \text{ part} = \frac{336}{8} = \text{PKR } 42$

5 parts = 42×5 = PKR 210

3 parts = 42×3 = PKR 126

∴ PKR 210 : PKR 126

(e) 12 parts = PKR 336

$1 \text{ part} = \frac{336}{12} = \text{PKR } 28$

5 parts = 28×5 = PKR 140

7 parts = 28×7 = PKR 196

∴ PKR 140 : PKR 196

(f) 14 parts = PKR 336

$1 \text{ part} = \frac{336}{14} = \text{PKR } 24$

5 parts = 24×5 = PKR 120

9 parts = 24×9 = PKR 216

∴ PKR 120 : PKR 216

(g) 24 parts = PKR 336

$1 \text{ part} = \frac{336}{24} = \text{PKR } 14$

7 parts = 14×7 = PKR 98

17 parts = 14×17 = PKR 238

∴ PKR 98 : PKR 238

(h) 21 parts = PKR 336

$1 \text{ part} = \frac{336}{21} = \text{PKR } 16$

8 parts = 16×8 = PKR 128

13 parts = 16×13 = PKR 208

∴ PKR 128 : PKR 208

(i) 21 parts = PKR 336

$1 \text{ part} = \frac{336}{21} = \text{PKR } 16$

10 parts = 16×10 = PKR 160

11 parts = 16×11 = PKR 176

∴ PKR 160 : PKR 176

(j) 24 parts = PKR 336

$1 \text{ part} = \frac{336}{24} = \text{PKR } 14$

11 parts = 14×11 = PKR 154

13 parts = 14×13 = PKR 182

∴ PKR 154 : PKR 182

5. **(a)** Convert PKR 1 to paisas.

PKR 1 = 100 paisas

45 paisas : 100 paisas

$= \frac{45}{100}$

$= \frac{9}{20}$

∴ 45 paisas : PKR 1 = 9 : 20

(b) Convert 1.25 m to cm.

$1.25 \text{ m} = 1.25 \times 100 = 125 \text{ cm}$

$25 \text{ cm} : 125 \text{ cm}$

$= \frac{25}{125}$

$= \frac{1}{5}$

$\therefore 25 \text{ cm} : 1.25 \text{ m} = 1 : 5$

(c) Convert 0.25 km to m.

$0.25 \text{ km} = 0.25 \times 1000 = 250 \text{ m}$

$250 \text{ m} : 75 \text{ m}$

$= \frac{250}{75}$

$= \frac{10}{3}$

$\therefore 0.25 \text{ km} : 75 \text{ m} = 10 : 3$

(d) Convert 0.2 kg to g.

$0.2 \text{ kg} = 0.2 \times 1000 = 200 \text{ g}$

$200 \text{ g} : 40 \text{ g}$

$= \frac{200}{40}$

$= \frac{5}{1}$

$\therefore 0.2 \text{ kg} : 40 \text{ g} = 5 : 1$

(e) Convert 1 hour to minutes.

$1 \text{ hour} = 60 \text{ minutes}$

$35 \text{ min} : 60 \text{ min}$

$= \frac{35}{60}$

$= \frac{7}{12}$

$\therefore 35 \text{ minutes} : 1 \text{ hour} = 7 : 12$

(f) Convert 2 cm to mm.

$2 \text{ cm} = 2 \times 10 = 20 \text{ mm}$

$15 \text{ mm} : 20 \text{ mm}$

$= \frac{15}{20}$

$= \frac{3}{4}$

$\therefore 15 \text{ mm} : 2 \text{ cm} = 3 : 4$

(g) Convert 3.2 hours to minutes.

$3.2 \text{ hours} = 3.2 \times 60 = 192 \text{ minutes}$

$192 \text{ min} : 72 \text{ min}$

$= \frac{192}{72}$

$= \frac{8}{3}$

$\therefore 3.2 \text{ hours} : 72 \text{ minutes} = 8 : 3$

(h) Convert $\frac{7}{200}\ l$ to cm^3.

$\frac{7}{200}\ l = \frac{7}{200} \times 1000 = 35 \text{ cm}^3$

$35 \text{ cm}^3 : 105 \text{ cm}^3$

$= \frac{35}{105}$

$= \frac{1}{3}$

$\therefore \frac{7}{200}\ l : 105 \text{ cm}^3 = 1 : 3$

6. **(a)** $57 : 19 : 133$

$57 \div 19 : 19 \div 19 : 133 \div 19$

$3 : 1 : 7$

(b) $64 : 96 : 224$

$64 \div 32 : 96 \div 32 : 224 \div 32$

$2 : 3 : 7$

(c) $108 : 36 : 60$

$108 \div 6 : 36 \div 6 : 60 \div 6$

$18 : 6 : 10$

$18 \div 2 : 6 \div 2 : 10 \div 2$

$9 : 3 : 5$

(d) $644 : 476 : 140$

$644 \div 28 : 476 \div 28 : 140 \div 28$

$23 : 17 : 5$

(e) $665 : 1995 : 1330$

$665 \div 35 : 1995 \div 35 : 1330 \div 35$

$19 : 57 : 38$

$19 \div 19 : 57 \div 19 : 38 \div 19$

$1 : 3 : 2$

(f) $1015 : 350 : 455$

$1015 \div 35 : 350 \div 35 : 455 \div 35$

$29 : 10 : 13$

7. **(a)** $3 : 9 = 4 : a$

$\frac{3}{9} = \frac{4}{a}$ (express ratios as fractions)

$3a = 36$

$a = 12$

(b) $4 : 3 = a : 6$

$\frac{4}{3} = \frac{a}{6}$ (express ratios as fractions)

$3a = 24$

$a = 8$

(c) $5 : 11 = 10 : a$

$\frac{5}{11} = \frac{10}{a}$

$5a = 110$

$a = 22$

(d) $12 : 25 = a : 5$

$$\frac{12}{25} = \frac{a}{5}$$

$$25a = 60$$

$$a = \frac{60}{25} = 2\frac{2}{5}$$

(e) $14 : 9 = 7 : a$

$$\frac{14}{9} = \frac{7}{a}$$

$$14a = 63$$

$$a = 4.5 \text{ or } 4\frac{1}{2}$$

(f) $a : 5.7 = 8 : 12$

$$\frac{a}{5.7} = \frac{8}{12}$$

$$12a = 45.6$$

$$a = 3.8 \text{ or } 3\frac{4}{5}$$

8. (i) Convert 1.68 cm to cm.

1.68 m = 1.68 × 100 = 168 cm

168 cm : 105 cm

$$= \frac{168}{105}$$

$$= \frac{8}{5}$$

∴ The ratio of Rizwan's height to his brother's height is 8 : 5.

(ii) Total height of the boys (in cm)

= 168 + 105 = 273 cm

1.68 m : 273 cm

168 cm : 273 cm

$$= \frac{168}{273}$$

$$= \frac{8}{13}$$

∴ The ratio of Rizwan's height to the total height of both boys is 8 : 13.

9. Total number of parts = 126 + 42 = 168 parts

(i)

Total number of parts	:	Number of parts of pure gold
168	:	126
168 ÷ 42	:	126 ÷ 42
4	:	3

(ii)

Total number of parts	:	Number of parts of alloy B
168	:	42
168 ÷ 42	:	42 ÷ 42
4	:	1

Alloy B : Pure Gold

1 : 3

10. (a) For the ratio 1 : 2 : 6,

9 parts = PKR 180

$$1 \text{ part} = \frac{180}{9} = \text{PKR } 20$$

6 parts = 20 × 6 = PKR 120

∴ The smallest share is PKR 20 and the largest share is PKR 120.

(b) For the ratio 1 : 4 : 7,

12 parts = PKR 180

$$1 \text{ part} = \frac{180}{12} = \text{PKR } 15$$

7 parts = 15 × 7 = PKR 105

∴ The smallest share is PKR 15 and the largest share is PKR 105.

(c) For the ratio 2 : 3 : 5,

10 parts = PKR 180

$$1 \text{ part} = \frac{180}{10} = \text{PKR } 18$$

2 parts = 18 × 2 = PKR 36

5 parts = 18 × 5 = PKR 90

∴ The smallest share is PKR 36 and the largest share is PKR 90.

(d) For the ratio 2 : 13 : 5,

20 parts = PKR 180

$$1 \text{ part} = \frac{180}{20} = \text{PKR } 9$$

2 parts = 9 × 2 = PKR 18

13 parts = 9 × 13 = PKR 117

∴ The smallest share is PKR 18 and the largest share is PKR 117.

(e) For the ratio 3 : 1 : 11,

15 parts = PKR 180

$$1 \text{ part} = \frac{180}{15} = \text{PKR } 12$$

11 parts = 12 × 11 = PKR 132

∴ The smallest share is PKR 12 and the largest share is PKR 132.

(f) For the ratio 4 : 11 : 3,

18 parts = PKR 180

$$1 \text{ part} = \frac{180}{18} = \text{PKR } 10$$

3 parts = 10 × 3 = PKR 30

11 parts = 10 × 11 = PKR 110

∴ The smallest share is PKR 30 and the largest share is PKR 110.

11. (a) 7 parts = PKR 84

$1 \text{ part} = \frac{84}{7} = \text{PKR } 12$

18 parts = 12 × 18 = PKR 216

∴ Largest part is PKR 216.

Total sum = (15 + 18 + 7) × 12 = PKR 480

(b) 7 parts = PKR 133

$1 \text{ part} = \frac{133}{7} = \text{PKR } 19$

18 parts = 19 × 18 = PKR 342

∴ Largest part is PKR 342.

Total sum = (15 + 18 + 7) × 19 = PKR 760

(c) 7 parts = PKR 301

$1 \text{ part} = \frac{301}{7} = \text{PKR } 43$

18 parts = 43 × 18 = PKR 774

∴ Largest part is PKR 774.

Total sum = (15 + 18 + 7) × 43 = PKR 1720

(d) 7 parts = PKR 3990

$1 \text{ part} = \frac{3990}{7} = \text{PKR } 570$

18 parts = 570 × 18 = PKR 10 260

∴ Largest part is PKR 10 260.

Total sum = (15 + 18 + 7) × 570 = PKR 22 800

12. (a) 11 parts = 187°

$1 \text{ part} = \frac{187}{11} = 17°$

7 parts = 17 × 7 = 119°

Angle D = 360 – 187 = 173°

Ratio of angle C to angle D = 119 : 173

(b) 11 parts = 242°

$1 \text{ part} = \frac{242}{11} = 22°$

7 parts = 22 × 7 = 154°

Angle D = 360 – 242 = 118°

Ratio of angle C to angle D = 154 : 118

(c) 11 parts = 275°

$1 \text{ part} = \frac{275}{11} = 25°$

7 parts = 25 × 7 = 175°

Angle D = 360 – 275 = 85°

Ratio of angle C to angle D = 175 : 85

= 35 : 17

13. (a) Rate $= \frac{350}{40} = \frac{35}{4} = 8.75$ km/l

(b) Rate $= \frac{120}{8} = \text{PKR } 15/\text{hour}$

(c) Rate $= \frac{82 \times 100}{300} = \frac{82}{3} = 27\frac{1}{3}$ paisas/unit

(d) Rate $= \frac{320}{8} = 40$ words/min

(e) Rate $= \frac{60}{12} = \text{PKR } 5/\text{tile}$

(f) Rate $= \frac{1760}{15} = 117\frac{1}{3}$ paisas/min

14. (i) Cost of 1 m^2 of flooring $= \frac{\text{PKR } 36}{20} = \text{PKR } 1.80$

(ii) Cost of 55 m^2 of flooring

= PKR 1.80 × 55 = PKR 99

(iii) Area of flooring for a cost of PKR 1

$= \frac{20}{36} = \frac{5}{9}$ m^2

Area of flooring for the cost of PKR 63

$= \frac{5}{9} \times \text{PKR } 63 = 35$ m^2

15. Amount required to travel a distance of 50 km

= PKR 1.35 × 50 = PKR 67.50

Amount that each child will have to pay $= \frac{\text{PKR } 67.50}{54}$

= PKR 1.25

16. Convert 75 cm to m.

75 cm = 75 ÷ 100 = 0.75 m

Area of rectangular brass sheet = 1.5 × 0.75

= 1.125 m^2

Area of 1 kg of brass sheet $= \frac{1.125}{7.2}$

= 0.156 25 m^2

Area of 12.8 kg of brass sheet = 0.156 25 × 12.8

= 2 m^2

17. Time required for one man to finish the project

= 45 × 8

= 360 hours

Time required for (45 – 5) = 40 men to finish the project $= \frac{360}{40}$

= 9 hours

Intermediate

18. (a) 16 parts = PKR 160

$$1 \text{ part} = \frac{160}{16} = \text{PKR } 10$$

9 parts = PKR 10 × 9 = PKR 90

Difference between the largest share and the smallest share

= PKR 90 – PKR 10

= PKR 80

(b) 20 parts = PKR 160

$$1 \text{ part} = \frac{160}{20} = \text{PKR } 8$$

2 parts = PKR 8 × 2 = PKR 16

13 parts = PKR 8 × 13 = PKR 104

Difference between the largest share and the smallest share

= PKR 104 – PKR 16

= PKR 88

(c) 40 parts = PKR 160

$$1 \text{ part} = \frac{160}{40} = \text{PKR } 4$$

5 parts = PKR 4 × 5 = PKR 20

22 parts = PKR 4 × 22 = PKR 88

Difference between the largest share and the smallest share

= PKR 88 – PKR 20

= PKR 68

(d) 80 parts = PKR 160

$$1 \text{ part} = \frac{160}{80} = \text{PKR } 2$$

11 parts = PKR 2 × 11 = PKR 22

37 parts = PKR 2 × 37 = PKR 74

Difference between the largest share and the smallest share

= PKR 74 – PKR 22

= PKR 52

19. (a) $X : Y = 2 : 3$ $\quad Y : Z = 5 : 4$

$= 10 : 15$ $\quad = 15 : 12$

$\therefore X : Z = 10 : 12 = 5 : 6$

(b) $X : Y = 5 : 7$ $\quad Y : Z = 13 : 10$

$= 65 : 91$ $\quad = 91 : 70$

$\therefore X : Z = 65 : 70 = 13 : 14$

(c) $X : Y = 7 : 3$ $\quad Y : Z = 11 : 21$

$= 77 : 33$ $\quad = 33 : 63$

$\therefore X : Z = 77 : 63 = 11 : 9$

(d) $X : Y = 8 : 15$ $\quad Y : Z = 21 : 32$

$= 56 : 105$ $\quad = 105 : 160$

$\therefore X : Z = 56 : 160 = 7 : 20$

20. Rice *B* is sold at PKR 6.90 for 5 kg. Thus it is sold at PKR 13.80 for 10 kg.

Ratio of prices of rice *A* and *B*

= PKR 9.20 : PKR 13.80

= 920 : 1380

= 2 : 3

21. $A : B = 8 : 3$ $\quad A : C = 5 : 12$

$= 40 : 15$ $\quad = 40 : 96$

The ratio of salaries *A*, *B* and *C*

= 40 : 15 : 96

22. Height of the hall $= \frac{28}{7} \times 6 = 24$ m

Ratio of its breadth to its height

= 21 : 24

= 7 : 8

23. (i) Amount each tourist spends for 4 days

$$= \frac{\text{PKR } 3600}{9} = \text{PKR } 400$$

Cost of staying in the hotel for one day

$$= \frac{\text{PKR } 400}{4} = \text{PKR } 100$$

Cost of staying in the hotel for 6 days

= PKR 100 × 6 = PKR 600

Amount 15 tourists spend for staying in the hotel for 6 days

= PKR 600 × 15

= PKR 9000

(ii) Amount each tourist spends $= \frac{\text{PKR } 3000}{10}$

= PKR 300

Number of days each tourist can stay in the hotel

$$= \frac{\text{PKR } 300}{\text{PKR } 100} = 3$$

24. (i) Charges due to the number of calls

= 493 × PKR 0.1605

= PKR 79.1265

Total charges for the month

= PKR 82.93 + PKR 79.1265

= PKR 162.06 (to the nearest cent)

(ii) Charges due to calls = PKR 93.523 – PKR 82.93

= PKR 10.593

Number of calls made $= \frac{\text{PKR } 10.593}{\text{PKR } 0.1605} = 66$

She made 66 calls.

25. No. of hours 1 man will take to complete 1200 m
$= 8 \times 20 \times 50$
$= 8000$ h
No. of hours 1 man will take to complete 1800 m
$= \frac{1800}{1200} \times 8000$
$= 12\,000$ h
No. of men needed to complete the work on time
$= \frac{12\,000}{10 \times 10}$
$= 120$
Additional number of men to be employed
$= 120 - 60$
$= 70$

26. (i) Amount of time to work on the project per day
$= 8.5 \times 4$
$= 34$ h
Time required to finish the work
$= \frac{272}{34} = 8$ days
It will take 8 days for 4 men to finish the work.

(ii) Amount to be paid to the men per day
= PKR $8.50 \times 8.5 \times 4$
= PKR 289
Total amount to be paid for the whole project
$= 8 \times$ PKR 289
= PKR 2312

(iii) Let the number of overtime hours needed to complete the project in 4 days by each worker be x.
$5[4(8.5 + x)] = 272$
$5(34 + 4x) = 272$
$170 + 20x = 272$
$20x = 272 - 170 = 102$
$x = 5.1$
The number of overtime hours is 5.1 h.

(iv) Overtime hourly rate
$= 1.5 \times$ PKR 8.50
= PKR 12.75
Total amount to be paid to the 4 men if the project is to be completed in 5 days
$= 5\{4[(8.5 \times$ PKR $8.50) + (5.1 \times$ PKR $12.75)]\}$
= PKR 2745.50

27. Distance travelled by the wheel $= 765 \times 2.8$
$= 2142$ m
Number of revolutions made by the wheel to travel a distance of 2142 m
$= \frac{2142}{1.7}$
$= 1260$ times

Advanced

28. $\frac{a - 2b}{10} = \frac{b}{6}$
$6(a - 2b) = 10b$
$6a - 12b = 10b$
$6a = 10b + 12b$
$6a = 22b$
$\frac{6a}{b} = 22$
$\frac{a}{b} = \frac{22}{6} = \frac{11}{3}$
The ratio of $a : b = 11 : 3$.

New Trend

29. (a) (i) $\frac{7}{3} = \frac{\text{Number of sedans}}{180}$
Number of sedans $= \frac{7}{3} \times 180 = 420$

(ii) Number of vehicles altogether
$= \frac{180}{3} \times (7 + 3 + 2)$
$= 720$

(b) Blue : Black : White
$3 : 5$
$\times 7$: $6 : 7$
$\times 5$, $\times 5$
$= 21 : 30 : 35$
∴ Blue sedan : black sedan : white sedan
$= 21 : 30 : 35$

30. Faiza gets $13 - 7 = 6$ parts more than Nadia.

(a) 6 parts = PKR 78
1 part $= \frac{78}{6} =$ PKR 13
12 parts = PKR $13 \times 12 =$ PKR 156

(b) 6 parts = PKR 126
1 part $= \frac{126}{6} =$ PKR 21
12 parts = PKR $21 \times 12 =$ PKR 252

(c) 6 parts = PKR 360
1 part $= \frac{360}{6} =$ PKR 60
12 parts = PKR $60 \times 12 =$ PKR 720

(d) 6 parts = PKR 540
1 part $= \frac{540}{6} =$ PKR 90
12 parts = PKR $90 \times 12 =$ PKR 1080

31. (i) Distance travelled on 1 litre of petrol

$= \frac{128}{12}$

$= 10\frac{2}{3}$ km

Distance travelled on 30 litres of petrol

$= 10\frac{2}{3} \times 30$

= 320 km

(ii) Amount of petrol required to travel a distance of 1 km

$= \frac{12}{128}$ litres

Amount of petrol required to travel a distance of 15 000 km

$= \frac{12}{128} \times 15\,000$

= 1406.25 litres

Amount the car owner has to pay

= 1406.25 × PKR 2.03

= PKR 2854.69 (to the nearest paisa)

Chapter 8 Perimeter and Area of Plane Figures

Basic

1. **(a)** $7.3\ cm^2 = 7.3 \times 10 \times 10$
$= 730\ mm^2$

(b) $4.65\ m^2 = 4.65 \times 10\,000$
$= 46\,500\ cm^2$

(c) $3650\ mm^2 = 3650 \div 100$
$= 36.5\ cm^2$

(d) $200\,000\ cm^2 = 200\,000 \div 10\,000$
$= 20\ m^2$

(e) $50\,000\ mm^2 = 50\,000 \div 100 \div 10\,000$
$= 0.05\ m^2$

2. **(a)** Breadth of rectangle $= \frac{48}{8}$
$= 6$ cm
Perimeter of rectangle $= 2(6 + 8)$
$= 28$ cm

(b) Breadth of rectangle $= \frac{0.9}{1.2}$
$= 0.75$ m
Perimeter of rectangle $= 2(0.75 + 1.2)$
$= 3.9$ m

(c) Length of rectangle $= \frac{1.76}{0.8}$
$= 2.2$ cm
Perimeter of rectangle $= 2(0.8 + 2.2)$
$= 6$ cm

3. Perimeter of square $= 4 \times$ length of square
$48 = 4 \times$ length of square
Length of square $= \frac{48}{4}$
$= 12$ cm
$\therefore$ Area of square $= 12 \times 12$
$= 144\ cm^2$

4. **(a)** **(i)** Perimeter of figure
$= 2 + 3 + 1 + 2 + 1 + 1$
$= 10$ cm

(ii) Area of figure
$= (2 \times 1) + (2 \times 1)$
$= 2 + 2$
$= 4\ cm^2$

(b) **(i)** Perimeter of figure
$= 3 + 9 + 3 + 3 + 3 + 3 + 3 + 3$
$= 30$ cm

(ii) Area of figure
$= (9 \times 3) + (3 \times 3)$
$= 27 + 9$
$= 36\ cm^2$

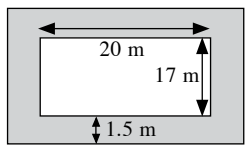
(c) **(i)** Perimeter of figure
$= 12 + 6 + 6 + 6 + 12 + 6 + 6 + 6$
$= 60$ cm

(ii) Area of figure
$= 2(12 \times 6)$
$= 2(72)$
$= 144\ cm^2$

(d) **(i)** Perimeter of figure
$= 14 + 7 + 7 + 7 + 14 + 7 + 7 + 7$
$= 70$ cm

(ii) Area of figure
$= 2(14 \times 7)$
$= 2(98)$
$= 196\ cm^2$

5. Length of the pool with the walkway
$= 20 + 1.5 + 1.5$
$= 23$ m



Breadth of the pool with the walkway
$= 17 + 1.5 + 1.5$
$= 20$ m
Area of pool with walkway
$= 23 \times 20$
$= 460\ m^2$
Area of the swimming pool
$= 20 \times 17$
$= 340\ m^2$
Area of walkway $= 460 - 340$
$= 120\ m^2$

6.

	Base	Height	Area
(a)	10 cm	12 cm	$10 \times 12 = 120\ cm^2$
(b)	$100 \div 5$ $= 20$ m	5 m	$100\ m^2$
(c)	5.2 mm	$50.96 \div 5.2$ $= 9.8$ mm	$50.96\ mm^2$

7.

	Parallel side 1	Parallel side 2	Height	Area
(a)	5 cm	11 cm	4 cm	$\frac{1}{2}(5+11)\times 4$ $= 32\ \text{cm}^2$
(b)	6 m	14 m	$65 \div \left[\frac{1}{2}(6+14)\right]$ $= 6.5\ \text{m}$	$65\ \text{m}^2$
(c)	2 mm	$(34.65 \div 8.25) \times 2 - 2$ $= 6.4\ \text{mm}$	8.25 mm	$34.65\ \text{mm}^2$

8. **(a)** The figure shown is a trapezium.

Area of the trapezium

$= \frac{1}{2}(11 + 13) \times 9$

$= 108\ \text{cm}^2$

(b) The figure shown is a parallelogram.

Area of parallelogram

$= 16 \times 9$

$= 144\ \text{cm}^2$

(c) If we rearrange the figure, it turns out to be a parallelogram

Area of the figure

$= 18 \times \left(\frac{1}{2} \times 16\right)$

$= 144\ \text{cm}^2$

(d) The figure is a rhombus and it is a special case of parallelogram.

Area of rhombus

$= 32 \times \left(\frac{1}{2} \times 18\right)$

$= 288\ \text{cm}^2$

(e) The figure is a trapezium.

Area of trapezium

$= \frac{1}{2}(8.3 + 11.7) \times 7.2$

$= 72\ \text{cm}^2$

(f) The figure is a trapezium and a rectangle.

Area of figure

$= \left[\frac{1}{2}(9 + 26) \times (32 - 10)\right] + (26 \times 10)$

$= 385 + 260$

$= 645\ \text{cm}^2$

(g) The figure is made up of two trapeziums.

Area of figure

$= \left[\frac{1}{2}(9 + 23) \times 10\right] + \left[\frac{1}{2}(9 + 17) \times 7\right]$

$= 160 + 91$

$= 251\ \text{cm}^2$

9. **(a)** Area of the figure

$= \frac{1}{2} \times 11 \times 14$

$= 77\ \text{cm}^2$

Area of figure $= \frac{1}{2} \times k \times 16$

$77 = \frac{1}{2} \times k \times 16$

$77 = 8k$

$\therefore k = \frac{77}{8} = 9\frac{5}{8}$

(b) Area of parallelogram $= 16 \times x$

$144 = 16x$

$x = 9$

(c) Area of $ABCD = \frac{1}{2}(18 + 24) \times h$

$273 = \frac{1}{2}(18 + 24) \times h$

$273 = 21h$

$h = 13$

(d) Area of $ABCD = \frac{1}{2}(32 + y) \times 24$

$912 = \frac{1}{2}(32 + y) \times 24$

$38 = \frac{1}{2}(32 + y)$

$76 = 32 + y$

$y = 76 - 32$

$= 44$

(e) Area of trapezium $= \frac{1}{2}(27 + 37) \times x$

$480 = \frac{1}{2}(27 + 37) \times x$

$960 = 64x$

$x = 15$

10. Let the perpendicular height be h cm.

Area of parallelogram $= (4 + 3) \times h$

$35 = 7h$

$h = 5$

Area of $\triangle PQT = \frac{1}{2} \times 4 \times 5$

$= 10\ \text{cm}^2$

11. (i) Area of parallelogram $ABCD$
$= 28 \times 22$
$= 616 \text{ cm}^2$

(ii) Area of parallelogram $ABCD = 18 \times AB$
$616 = (18 \times AB) \text{ cm}^2$
$AB = 34\frac{2}{9} \text{ cm}$
Perimeter of parallelogram
$= 2\left(22 + 34\frac{2}{9}\right)$
$= 112\frac{4}{9} \text{ cm}$

12. Let the length of the other parallel side be y cm.

Area of trapezium $= \frac{1}{2}(6 + y) \times 5$

$$45 = \frac{1}{2}(6 + y) \times 5$$
$$90 = 5(6 + y)$$
$$18 = 6 + y$$
$$y = 18 - 6$$
$$= 12$$

The length of the other parallel side is 12 cm.

13. (a) Area of shaded region
$= \left(\frac{1}{2} \times 4.6 \times 8\right) + \left(\frac{1}{2} \times 6.5 \times 8\right)$
$= 18.4 + 26$
$= 44.4 \text{ cm}^2$

(b) Area of shaded region
$= (3.5 \times 4.6) + [(3.8 + 3.5 + 3.7) \times 3.4]$
$= 16.1 + 11 \times 3.4$
$= 16.1 + 37.4$
$= 53.5 \text{ cm}^2$

(c) Area of rectangle
$= 13 \times 11$
$= 143 \text{ cm}^2$
Area of triangle with perpendicular height of 5 cm
$= \frac{1}{2} \times 11 \times 5$
$= 27.5 \text{ cm}^2$
Area of triangle with perpendicular height of 4 cm
$= \frac{1}{2} \times 11 \times 4$
$= 22 \text{ cm}^2$
Area of shaded region
$= 143 - 27.5 - 22$
$= 93.5 \text{ cm}^2$

Intermediate

14. (a) Let the length of the square be n cm.
$\therefore n^2 = 900$
Thus $n = \sqrt{900} = 30$ cm
Perimeter of square $= 4 \times 30$
$= 120$ cm

(b) Let the length of the square be x cm.
$12.8 = 4x$
$x = 3.2$
Area of the square $= (3.2)^2$
$= 10.24 \text{ cm}^2$

15. (a) (i) Let the breadth of the rectangle be y cm.
$2[y + (y + 8)] = 80$
$y + y + 8 = 40$
$2y = 40 - 8$
$2y = 32$
$y = 16$
The length of the rectangle is $(16 + 8)$
$= 24$ cm.

(ii) Area of the rectangle
$= 16 \times 24$
$= 384 \text{ cm}^2$

(b) Let the length of the rectangle be x m.
$0.464 \times x = 11.6$
$x = 25$ m
Perimeter of rectangle
$= 2(25 + 0.464)$
$= 50.928$ m

(c) Let the breadth of the rectangle be y cm.
Then the length of the rectangle is $(3y)$ cm.
Perimeter of rectangle $= 2(3y + y)$ cm
$1960 = 2(3y + y)$
$980 = 4y$
$\therefore y = 245$
The breadth is 245 cm and the length is 735 cm.
Area of the rectangle
$= 735 \times 245$
$= 180\,075 \text{ cm}^2$
$= 180\,075 \div 10\,000$
$= 18.0075 \text{ m}^2$

(d) Let the breadth of the rectangle be x cm.
Then the length of the rectangle is $2x$ cm.
Circumference of the wire

$= 2 \times 3.142 \times \frac{35}{2}$

$= 3.142 \times 35$

$= 109.97$ cm

Circumference of the wire is the perimeter of the rectangle.

$109.97 = 2(x + 2x)$

$54.985 = x + 2x$

$3x = 54.985$

$x = 18.328\ 33$ (to 5 d.p.)

Area of the rectangle

$= 18.328\ 33 \times 2(18.328\ 33)$

$= 672$ cm^2 (to 3 s.f.)

16. (a) Area of $\triangle ACD = \frac{1}{2} \times DC \times AB$

$8.4 = \frac{1}{2} \times 4 \times AB$

$2AB = 8.4$

$AB = 4.2$ cm

(b) Area of $\triangle ABC$

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 6 \times 4.2$

$= 12.6$ cm^2

17. (a) The height from X to the length $PQ = \frac{10}{2}$

$= 5$ cm

Area of $\triangle PQX = \frac{1}{2} \times 16 \times 5$

$= 40$ cm^2

(b) Area of $\triangle PQR = \frac{1}{2} \times (16 \times 10)$

$= 80$ cm^2

Area of $\triangle QRX = 80 - 40$

$= 40$ cm^2

18. (a) Since $ABCD$ is a square, then

$3x = 22$

$x = 7\frac{1}{3}$

(b) Area of shaded region

= area of square $ABCD$ – area of $PQRC$

$403 = (22 \times 22) - y^2$

$y^2 = (22 \times 22) - 403$

$= 484 - 403$

$= 81$

$\therefore y = 9$

19. (a) (i) Perimeter of rectangle

$= 2[(3x + 4) + (4x - 13)]$

$94 = 2[(3x + 4) + (4x - 13)]$

$94 = 2[3x + 4x + 4 - 13]$

$94 = 2[7x - 9]$

$94 = 14x - 18$

$14x = 94 + 18$

$14x = 112$

$\therefore x = 8$

(ii) Length of rectangle $= 3 \times 8 + 4$

$= 28$ cm

Breadth of rectangle $= 4 \times 8 - 13$

$= 19$ cm

Area of rectangle $= 28 \times 19$

$= 532$ cm^2

(b) Area of trapezium

$= \frac{1}{2} \times [(x + 5) + (3x + 1)] \times 6$

$= 3[(x + 5) + (3x + 1)]$

$66 = 3[(x + 5) + (3x + 1)]$

$66 = 3[x + 3x + 5 + 1]$

$66 = 3[4x + 6]$

$66 = 12x + 18$

$12x = 66 - 18$

$12x = 48$

$\therefore x = 4$

20. (a) Number of slabs needed along its length

$= \frac{25 \times 100}{25}$

$= 100$

(b) Number of slabs needed along its row

$= \frac{12 \times 100}{25}$

$= 48$

(c) Area of rectangular courtyard

$= (25 \times 100) \times (12 \times 100)$

$= 3\ 000\ 000$ cm^2

Area of each slab

$= 25 \times 25$

$= 625$ cm^2

Number of slabs needed to pave the whole courtyard

$= \frac{3\ 000\ 000}{625}$

$= 4800$

(d) Total cost of paving the courtyard

= PKR 0.74×4800

= PKR 3552

21. (a) Let the length AB be h cm.

Area of quadrilateral $ABCD$

$= 8 \times h$

$= 8h \text{ cm}^2$

Area of quadrilateral $EFGH$

$= 10 \times h$

$= 10h \text{ cm}^2$

Area of $\triangle IJK$

$= \frac{1}{2} \times 14 \times h$

$= 7h \text{ cm}^2$

Ratio of area of $ABCD$ to area of $EFGH$ to area of $\triangle IJK$

$= 8h : 10h : 7h$

$= 8 : 10 : 7$

(b) Area of $\triangle IJK = 56$

$$\frac{1}{2} \times 14 \times h = 56$$

$$7h = 56$$

$$h = 8$$

The quadrilateral $LMNO$ is a trapezium.

Area of quadrilateral $LMNO$

$= \frac{1}{2} \times (3 + 17) \times 8$

$= 80 \text{ cm}^2$

Advanced

22. (a) Perimeter of triangle ABC

$= 2x + (x + 5) + (4x - 2)$

$= 2x + x + 4x + 5 - 2$

$= (7x + 3) \text{ cm}$

Perimeter of rectangle $PQRS$

$= 2[(7x - 10) + (2x + 1)]$

$= 2(7x + 2x - 10 + 1)$

$= 2(9x - 9) \text{ cm}$

$= 18(x - 1) \text{ cm}$

The equation is $1\frac{1}{2}(7x + 3) = 18(x - 1)$.

(b) $1\frac{1}{2}(7x + 3) = 18(x - 1)$

$$3(7x + 3) = 36(x - 1)$$

$$21x + 9 = 36x - 36$$

$$36x - 21x = 9 + 36$$

$$15x = 45$$

$$x = 3$$

Perimeter of triangle $ABC = 7 \times 3 + 3$

$= 24 \text{ cm}$

Area of triangle ABC

$= \frac{1}{2} \times (2 \times 3) \times (3 + 5)$

$= \frac{1}{2} \times 6 \times 8$

$= 24 \text{ cm}^2$

(c) Area of rectangle $PQRS$

$= (2 \times 3 + 1) \times (7 \times 3 - 10)$

$= 7 \times 11$

$= 77 \text{ cm}^2$

Difference between the area of triangle ABC and the area of rectangle $PQRS$

$= 77 - 24$

$= 53 \text{ cm}^2$

Chapter 9 Volume and Surface Area of Cubes and Cuboids

Basic

1. **(a)** $6.2\ m^3 = 6.2 \times 100 \times 100 \times 100$
$= 6\ 200\ 000\ cm^3$

(b) $2.9\ m^3 = 2.9 \times 100 \times 100 \times 100$
$= 2\ 900\ 000\ cm^3$

(c) $35\ 000\ cm^3 = 35\ 000 \div 100 \div 100 \div 100$
$= 0.035\ m^3$

(d) $75\ cm^3 = 75 \div 100 \div 100 \div 100$
$= 0.000\ 075\ m^3$

(e) $97.8\ l = 97.8 \times 1000$
$= 97\ 800\ cm^3$

(f) $1\ cm^3 = 1\ ml$
$0.07\ cm^3 = 0.07\ ml$

2. **(a)** **(i)** Volume of cube $= 5^3$
$= 125\ cm^3$

(ii) Total surface area
$= 6l^2$
$= 6 \times 5^2 = 150\ cm^2$

(b) **(i)** Volume of cube $= 2.4^3$
$= 13.824\ cm^3$

(ii) Total surface area
$= 6 \times 2.4^2$
$= 34.56\ cm^2$

(c) **(i)** Volume of rectangular cuboid
$= 30 \times 25 \times 12$
$= 9000\ cm^3$

(ii) Total surface area of cuboid
$= 2[(30 \times 25) + (30 \times 12) + (25 \times 12)]$
$= 2[750 + 360 + 300]$
$= 2820\ cm^2$

(d) **(i)** Volume of rectangular cuboid
$= 1.2 \times 0.8 \times 0.45$
$= 0.432\ m^3$

(ii) Total surface area of cuboid
$= 2[(1.2 \times 0.8) + (1.2 \times 0.45) + (0.8 \times 0.45)]$
$= 2[0.96 + 0.54 + 0.36]$
$= 3.72\ m^2$

3. **(a)** The shape of the base is a cross.
Base area
$= (14 \times 14) - 4(5 \times 5)$
$= 96\ cm^2$
Volume of prism = base area × height
$= 96 \times 3$
$= 288\ cm^3$
Total surface area of solid
$= (2 \times 96) + 8(5 \times 3) + 4(3 \times 4)$
$= 192 + 120 + 48$
$= 360\ cm^2$

(b) The base is an inverted L-shape.
Base area $= (21 \times 15) - (9 \times 7)$
$= 315 - 63$
$= 252\ cm^2$
Volume of prism = base area × height
$= 252 \times 10$
$= 2520\ cm^3$
Total surface area of the solid
$= (252 \times 2) + 2(6 \times 10) + 2(7 \times 10) + (9 \times 10)$
$+ (21 \times 10) + 2(15 \times 10)$
$= 504 + 120 + 140 + 90 + 210 + 300$
$= 1364\ cm^2$

4. **(a)** Let the height of the room be h m.
Volume of room $= (12 \times 9 \times h)\ m^3$
$540 = 12 \times 9 \times h$
$h = 5$
$\therefore$ The height of the room is 5 m.

(b) Let the length of the box be n cm.
$60 = n \times 4 \times 2$
$\therefore n = 7.5$
The length of the box is 7.5 cm.

5. **(a)** Number of cubes that can be obtained along the length
$= 20 \div 4$
$= 5$
Number of cubes that can be obtained along the breadth
$= 16 \div 4$
$= 4$
Number of cubes that can be obtained along the height
$= 8 \div 4$
$= 2$
Therefore, the number of cubes that can be obtained
$= 5 \times 4 \times 2$
$= 40$

(b) Number of cubes that can be obtained along the length
$= 80 \div 4$
$= 20$
Number of cubes that can be obtained along the breadth
$= 25 \div 4$
≈ 6
Number of cubes that can be obtained along the height
$= 35 \div 4$
≈ 8
Therefore, the number of cubes that can be obtained
$= 20 \times 6 \times 8$
$= 960$

(c) Number of cubes that can be obtained along the length
$= 120 \div 4$
$= 30$
Number of cubes that can be obtained along the breadth
$= 85 \div 4$
≈ 21
Number of cubes that can be obtained along the height
$= 50 \div 4$
≈ 12
Therefore, the number of cubes that can be obtained
$= 30 \times 21 \times 12$
$= 7560$

6. Number of cubes that can be cut along the length
$= 420 \div 20$
$= 21$
Number of cubes that can be cut along the breadth
$= 140 \div 20$
$= 7$
Number of cubes that can be cut along the height
$= 120 \div 20$
$= 6$
Therefore, the number of cubes that can be cut
$= 21 \times 7 \times 6$
$= 882$
(Note: For questions 6 and 7, understand the difference between "cut" and "melt" and "recast".)

7. Total volume of water
$= 37 + 20$
$= 57 \text{ m}^3$
Let the depth of water in the trough be h m.
Volume of water $= 8 \times 3 \times h$
$= 24h \text{ m}^3$
$57 = 24h$
$\therefore h = 2.375$
The depth of the water, after 20 m^3 of water is added, is 2.375 m.

8. **(i)** Volume of air = volume of cuboid
$= 12 \times 7 \times 3$
$= 252 \text{ m}^3$

(ii) Number of students allowed staying in the dormitory
$= 252 \div 14$
$= 18$

Intermediate

9. Volume of block = volume of cube
$= (28)^3$
$= 21\,952 \text{ cm}^3$
Let one unit of the length of the block be y cm.
Then $(5y) \times (4y) \times (3y) = 21\,952$
$60y^3 = 21\,952$
$y^3 = 365\frac{13}{15}$
$y = 7.152$
Longest side of the cuboid
$= 5 \times 7.152$
$= 35.761$
$= 35.8$ cm (to 1 d.p.)

10. **(a)** **(i)** Convert 12 litres to cm^3.
$12\ l = 12 \times 1000 = 12\,000 \text{ cm}^3$
Height of water
= volume of water ÷ base area of tank
$= 12\,000 \div (40 \times 28)$
$= 10.714$
$= 10.7$ cm (to 3 s.f.)

(ii) Surface area in contact with the water
$= (40 \times 28) + 2[(40 \times 10.714) + (28 \times 10.714)]$
$= 1120 + 2[428.56 + 299.992]$
$= 2577.104$
$= 2580 \text{ cm}^2$ (to 3 s.f.)

(b) **(i)** Volume of tank
$= 65 \times 42 \times 38$
$= 103\,740 \text{ cm}^3$
Volume of each cylindrical cup
$= 3.142 \times (3.5)^2 \times 12$
$= 461.874 \text{ cm}^3$
Number of cups that can fill the tank
$= \frac{103740}{461.874}$
≈ 224.61
$= 224$ complete cups
(Note: The answer is not 225 as the question requires the number of **complete** cups.)

(ii) Volume of cup $= 224 \times 461.874$
$= 103\,460 \text{ cm}^3$
Volume of sugarcane left in the tank
$= 103\,740 - 103\,460$
$= 280 \text{ cm}^3$

11. (i) Let the length of the cube be l cm.

Total surface area of cube $= 6l^2$

$294 = 6l^2$

$6l^2 = 294$

$l = 7$

Volume of cube $= 7^3$

$= 343 \text{ cm}^3$

(ii) Convert 343 cm^3 to m^3.

$343 \text{ cm}^3 = 343 \div 100 \div 100 \div 100$

$= 3.43 \times 10^{-4} \text{ m}^3$

Density of solid cube

$= \dfrac{\text{mass}}{\text{volume}}$

$= \dfrac{1.47}{3.43 \times 10^{-4}}$

$= 4285.714$

$= 4290 \text{ kg/m}^3$ (to 3 s.f.)

12. (i) Volume of open rectangular tank

$= 110 \times 60 \times 40$

$= 264\,000 \text{ cm}^3$

Amount of liquid required to fill up the tank

$= \dfrac{3}{8} \times 264\,000$

$= 99\,000 \text{ cm}^3$

$= 99$ litres

(ii) Amount of time needed, in minutes, to fill up the tank

$= \dfrac{99}{5.5}$

$= 18$ minutes

(iii) Volume of liquid in tank, in m^3

$= 264\,400 \div 100 \div 100 \div 100$

$= 0.264 \text{ m}^3$

Mass of liquid in the whole tank

= density × volume of liquid in tank

$= 800 \times 0.264$

$= 211.2$ kg

Chapter 10 Basic Geometry

Basic

1. **(a)** $x° + 90° + 38° = 180°$ (adj. ∠s on a str. line)

$x° = 180° - 90° - 38°$

$= 52°$

$\therefore x = 52$

(b) $2x° + 80° = 180°$ (adj. ∠s on a str line)

$2x° = 180° - 80°$

$= 100°$

$x° = 50°$

$\therefore x = 50$

(c) $2x° + (5x - 9)° = 180°$ (adj. ∠s on a str. line)

$7x° - 9° = 180°$

$7x° = 189°$

$x° = 27°$

$\therefore x = 27$

(d) $(5x - 23)° + (7x - 13)° = 180°$ (adj. ∠s on a

$5x° + 7x° - 23° - 13° = 180°$ str. line)

$12x° - 36° = 180°$

$12x° = 180° + 36°$

$= 216°$

$x° = 18°$

$\therefore x = 18$

(e) $2x° + 90° + 3x° = 180°$ (adj. ∠s on a str. line)

$2x° + 3x° = 180° - 90°$

$5x° = 90°$

$x° = 18°$

$\therefore x = 18$

(f) $3x° + 4x° + 2x° = 180°$ (adj. ∠s on a str. line)

$9x° = 180°$

$x° = 20°$

$\therefore x = 20$

2. **(a)** $4x° + 3x° + 2x° = 180°$ (vert. opp. ∠s;

$9x° = 180°$ adj. ∠s on a str. line)

$x° = 20°$

$\therefore x = 20$

(b) $3x° + 49° + 62° = 180°$ (adj. ∠s on a str. line)

$3x° = 180° - 49° - 62°$

$3x° = 69°$

$x° = 23°$

$3x° + z° = 180°$ (adj. ∠s on a str. line)

$3(23°) + z° = 180°$

$69° + z° = 180°$

$z° = 180° - 69°$

$= 111°$

$y° + z° = 180°$ (adj. ∠s on a str. line)

$y° + 111° = 180°$

$y° = 180° - 111°$

$= 69°$

$\therefore x = 23$, $y = 69$ and $z = 111$

3. **(a)** $(3x + 34)° = (5x - 14)°$ (alt. ∠s, AB // CD)

$5x° - 3x° = 34° + 14°$

$2x° = 48°$

$x° = 24°$

$\therefore x = 24$

(b) $(7x - 12)° + (4x - 17)° = 180°$ (int. ∠s,

$7x° + 4x° - 12° - 17° = 180°$ AB // CD)

$11x° - 29° = 180°$

$11x° = 180° + 29°$

$11x° = 209°$

$x° = 19°$

$\therefore x = 19$

(c) $4x° + 5x° = 180°$ (alt. ∠s, adj. ∠s on a str. line)

$9x° = 180°$

$x° = 20°$

$\therefore x = 20$

(d) $(5x - 14)° + (3x - 10)° = 180°$ (alt. ∠s, adj. ∠s

$5x° + 3x° - 14° - 10° = 180°$ on a str. line)

$8x° - 24° = 180°$

$8x° = 180° + 24°$

$= 204°$

$x° = 25.5°$

$\therefore x = 25.5$

(e) $(5x - 15)° + (75 - x)° = 180°$ (vert. opp. ∠s,

$5x° - x° - 15° + 75° = 180°$ int. ∠s,

$4x° + 60° = 180°$ AB // CD)

$4x° = 180° - 60°$

$= 120°$

$x° = 30°$

$\therefore x = 30$

(f) $(3x + 40)° = (5x - 20)°$ (corr. ∠s, AB // CD)

$5x° - 3x° = 40° + 20°$

$2x° = 60°$

$x° = 30$

$(5x - 20)° = 2y°$ (vert. opp. ∠s)

$5 \times 30° - 20° = 2y°$

$2y° = 130°$

$y° = 65°$

$\therefore x = 30$ and $y = 65$

Intermediate

4. **(a)** $3x° + (7x - 21)° + (4x - 9)° = 180°$ (adj. ∠s on a str. line)

$3x° + 7x° + 4x° - 21° - 9° = 180°$

$14x° - 30° = 180°$

$14x° = 180° + 30°$

$= 210°$

$x° = 15°$

$\therefore x = 15$

(b) $\left(\frac{1}{3}x + 8\right)° + \left(\frac{3}{4}x - 18\right)° + \frac{1}{2}x°$

$= 180°$ (adj. ∠s on a str. line)

$\frac{1}{3}x° + \frac{3}{4}x° + \frac{1}{2}x° + 8° - 18° = 180°$

$1\frac{7}{12}x° = 180° + 10°$

$= 190°$

$x° = 120°$

$\therefore x = 120$

(c) $1.8x° + (2x + 12)° + x° = 180°$ (adj. ∠s on a str. line)

$1.8x° + 2x° + x° = 180° - 12°$

$4.8x° = 168°$

$x° = 35°$

$\therefore x = 35$

(d) $(0.5x + 14)° + (x + 15)° + (0.2x + 15)°$

$= 180°$ (adj. ∠s on a str. line)

$0.5x° + x° + 0.2x° + 14° + 15° + 15° = 180°$

$1.7x° + 44° = 180°$

$1.7x° = 136°$

$x° = 80°$

$\therefore x = 80$

5. **(a)** $3x° + (7x - 20)° = 180°$ (adj. ∠s on a str. line)

$3x° + 7x° = 180° + 20°$

$10x° = 200°$

$x° = 20°$

$3x° + y° = 180°$ (adj. ∠s on a str. line)

$3(20°) + y° = 180°$

$60° + y° = 180°$

$y° = 180° - 60° = 120°$

$\therefore x = 20$ and $y = 120$

(b) $(4x - 5)° + (8x - 41)° + 3x° + (3x + 10)°$

$= 360°$ (∠s at a point)

$4x° + 8x° + 3x° + 3x° - 5° - 41° + 10° = 360°$

$18x° - 36° = 360°$

$18x° = 360° + 36°$

$= 396°$

$x° = 22°$

$\therefore x = 22$

(c) $y° + 70° = 180°$ (adj. ∠s on a str. line)

$y° = 180° - 70°$

$= 110°$

$28° + (3x - 5)° + 70° = 180°$ (adj. ∠s on a str. line)

$3x° + 28° - 5° + 70° = 180°$

$3x° + 93° = 180°$

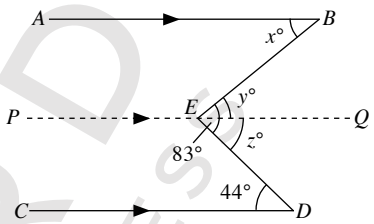
$3x° = 180° - 93°$

$= 87°$

$x° = 29°$

$\therefore x = 29$ and $y = 110$

6. **(a)** Draw a line PQ through E that is parallel to AB and CD.



$z° = 44°$ (alt. ∠s, PQ // CD)

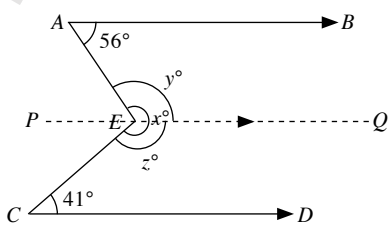
$y° = 83° - 44°$

$= 39°$

$x° = y° = 39°$ (alt. ∠s, PQ // AB)

$\therefore x = 39$

(b) Draw a line PQ through E that is parallel to AB and CD.



$41° + z° = 180°$ (int. ∠s, PQ // CD)

$z° = 180° - 41°$

$= 139°$

$56° + y° = 180°$ (int. ∠s, PQ // AB)

$y° = 180° - 56°$

$= 124°$

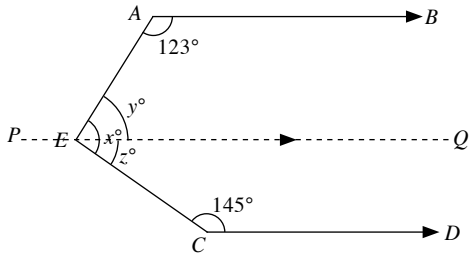
$x° = y° + z°$

$= 124° + 139°$

$= 263°$

$\therefore x = 263$

(c) Draw a line PQ through E that is parallel to AB and CD.



$123° + y° = 180°$ (int. $\angle$s, PQ // AB)

$y° = 180° - 123°$

$= 57°$

$145° + z° = 180°$ (int. $\angle$s, PQ // CD)

$z° = 180° - 145°$

$= 35°$

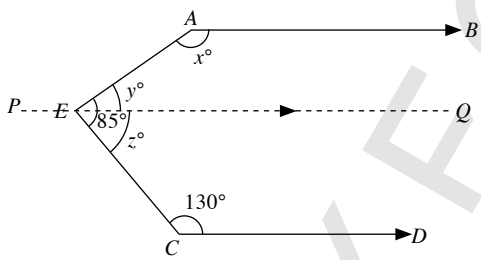
$x° = y° + z°$

$= 57° + 35°$

$= 92°$

$\therefore x = 92$

(d) Draw a line PQ through E that is parallel to AB and CD.



$130° + z° = 180°$ (int. $\angle$s, PQ // CD)

$z° = 180° - 130°$

$= 50°$

$z° + y° = 85°$

$y° = 85° - z°$

$= 85° - 50°$

$= 35°$

$y° + x° = 180°$ (int. $\angle$s, PQ // AB)

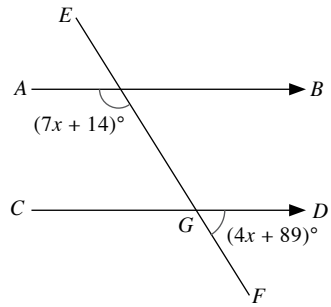
$x° = 180° - y°$

$= 180° - 35°$

$= 145°$

$\therefore x = 145$

(e)



$\angle CGE = (4x + 89)°$ (vert. opp. $\angle$s)

$(4x + 89)° + (7x + 14)° = 180°$ (int. $\angle$s, AB // CD)

$4x° + 7x° + 89° + 14° = 180°$

$11x° + 103° = 180°$

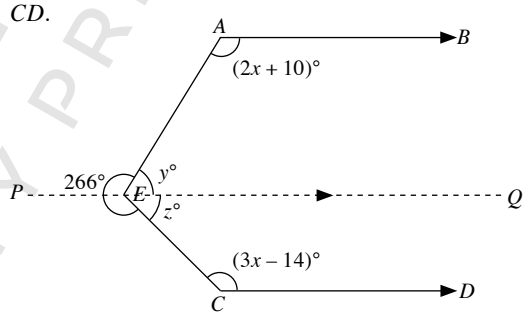
$11x° = 180° - 103°$

$= 77°$

$x° = 7°$

$\therefore x = 7$

(f) Draw a line PQ through E that is parallel to AB and CD.



$\angle AEC = 360° - 266° = 94°$ ($\angle$s at a point)

$y° + (2x + 10)° = 180°$

$y° = 180° - (2x + 10)°$

$= 180° - 2x° - 10°$

$= 170° - 2x°$

$z° + (3x - 14)° = 180°$

$z° = 180° - (3x - 14)°$

$= 180° - 3x° + 14°$

$= 194° - 3x°$

$y° + z° = 94°$

$170° - 2x° + 194° - 3x° = 94°$

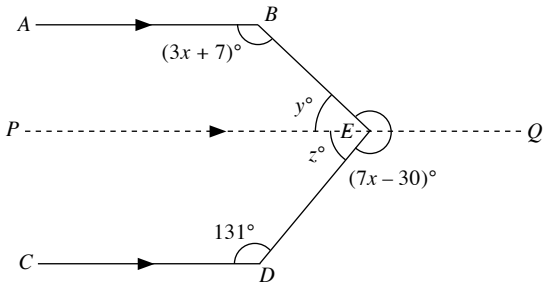
$2x° + 3x° = 170° + 194° - 94°$

$5x° = 270°$

$x° = 54°$

$\therefore x = 54$

(g) Draw a line PQ through E that is parallel to AB and CD.



$z° + 131° = 180°$ (int. $\angle$s, PQ // CD)

$z° = 180° - 131°$

$= 49°$

$y° + z° + (7x - 30)° = 360°$ ($\angle$s at a point)

$y° + 49° + (7x - 30)° = 360°$

$y° = 360° - 49° - (7x - 30)°$

$= 360° - 49° - 7x° + 30°$

$= 341° - 7x°$

$(3x + 7)° + y° = 180°$ (int. $\angle$s, PQ // AB)

$(3x + 7)° + 341° - 7x° = 180°$

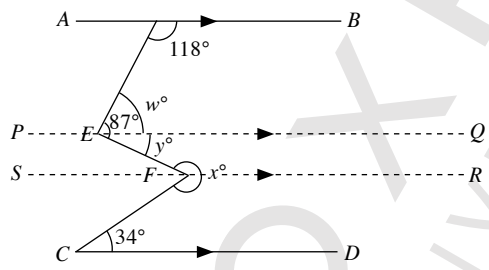
$3x° + 7° + 341° - 7x° = 180°$

$4x° = 168°$

$x° = 42°$

$\therefore x = 42$

(h) Draw a line PQ through E, and a line SR through F, that is parallel to AB and CD.



$118° + w° = 180°$ (int. $\angle$s, PQ // AB)

$w° = 180° - 118°$

$= 62°$

$w° + y° = 87°$

$y° = 87° - w°$

$= 87° - 62°$

$= 25°$

$\angle RFE + y° = 180°$ (int. $\angle$s, SR // PQ)

$\angle RFE = 180° - y°$

$= 180° - 25° = 155°$

$\angle CFR + 34° = 180°$ (int. $\angle$s, SR // CD)

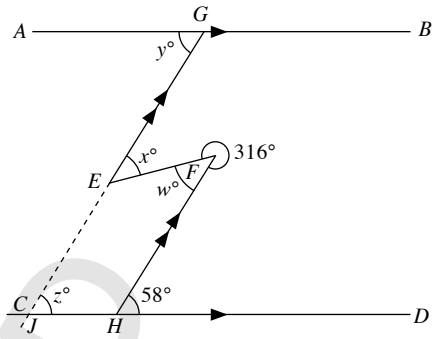
$\angle CFR = 180° - 34°$

$= 146°$

$x° = 155° + 146° = 301°$

$\therefore x = 301$

7. (a)



$w° + 316° = 360°$ ($\angle$s at a point)

$w° = 360° - 316° = 44°$

$w° = x°$ (alt. $\angle$s, EG // HF)

$x° = 44°$

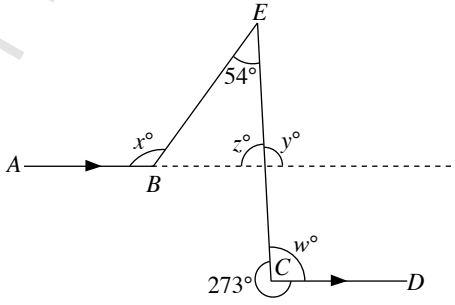
Extend the line EG to meet the line CD at J.

$z° = 58°$ (corr. $\angle$s, JG // HF)

$y° = 58°$ (alt. $\angle$s, AB // CD)

$\therefore x = 44$ and $y = 58$

(b) Extend the line AB to meet the line EC at F.



$w° + 273° = 360°$ ($\angle$s at a point)

$w° = 360° - 273° = 87°$

$y° = w° = 87°$ (corr. $\angle$s, AB // CD)

$z° + y° = 180°$ (adj. $\angle$s on a str. line)

$z° = 180° - y°$

$= 180° - 87°$

$= 93°$

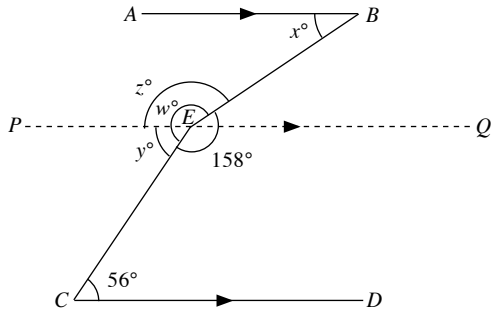
$x° = 54° + z°$ (ext. $\angle$ of $\triangle BEF$)

$= 54° + 93°$

$= 147°$

$\therefore x = 147$

(c) Draw a line PQ through E that is parallel to AB and CD.



$w° + 158° = 360°$ ($\angle$s at a point)

$w° = 360° - 158°$

$= 202°$

$y° = 56°$ (alt. $\angle$s, PQ // CD)

$z° + y° = w° = 202°$

$z° = 202° - y°$

$= 202° - 56°$

$= 146°$

$x° + z° = 180°$ (int. $\angle$s, PQ // AB)

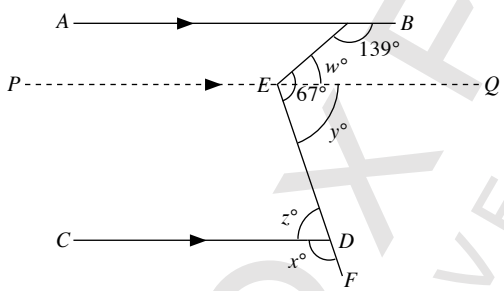
$x° = 180° - z°$

$= 180° - 146°$

$= 34°$

$\therefore x = 34$

(d) Draw a line PQ through E that is parallel to AB and CD.



$w° + 139° = 180°$ (int. $\angle$s, PQ // AB)

$w° = 180° - 139°$

$= 41°$

$w° + y° = 67°$

$y° = 67° - w°$

$= 67° - 41°$

$= 26°$

$z° = y° = 26°$ (alt. $\angle$s, PQ // CD)

$x° + z° = 180°$ (adj. $\angle$s on a str. line)

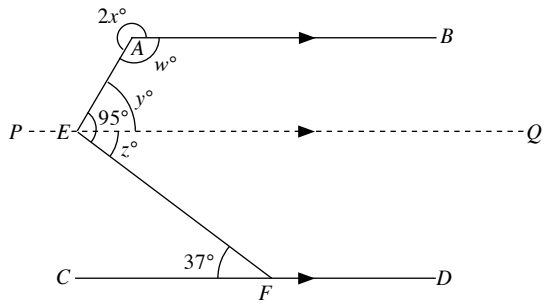
$x° = 180° - z°$

$= 180° - 26°$

$= 154°$

$\therefore x = 154$

(e) Draw a line PQ through E that is parallel to AB and CD.



$z° = 37°$ (alt. $\angle$s, PQ // CD)

$y° + z° = 95°$

$y° = 95° - z°$

$= 95° - 37°$

$= 58°$

$w° + y° = 180°$ (int. $\angle$s, PQ // AB)

$w° = 180° - y°$

$= 180° - 58°$

$= 122°$

$2x° + w° = 360°$ ($\angle$s at a point)

$2x° = 360° - w°$

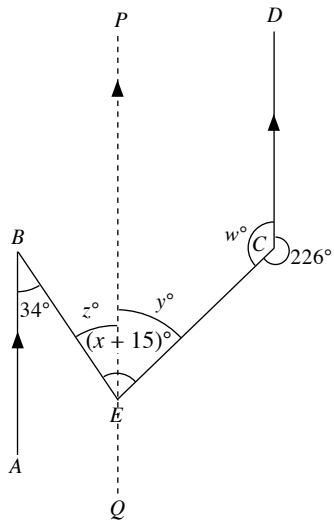
$= 360° - 122°$

$= 238°$

$x° = 119°$

$\therefore x = 119$

(f) Draw a line PQ through E that is parallel to AB and CD.



$w° + 226° = 360°$ ($\angle$s at a point)

$w° = 360° - 226°$

$= 134°$

$y° + w° = 180°$ (int. ∠s, PQ // DC)

$y° = 180° - w°$

$= 180° - 134°$

$= 46°$

$z° = 34°$ (alt. ∠s, PQ // BA)

$(x + 15)° = y° + z°$

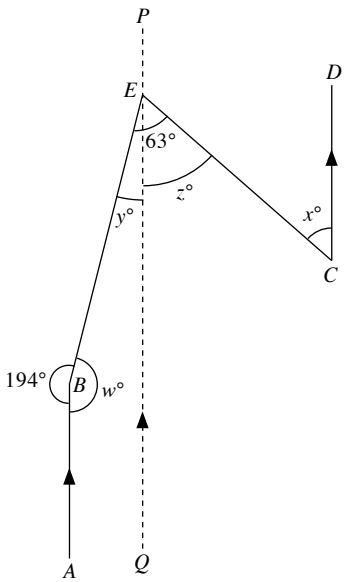
$(x + 15)° = 46° + 34° = 80°$

$x° = 80° - 15°$

$= 65°$

$\therefore x = 65$

(g) Draw a line PQ through E that is parallel to AB and CD.



$w° + 194° = 360°$ (∠s at a point)

$w° = 360° - 194°$

$= 166°$

$y° + w° = 180°$ (int. ∠s, PQ // BA)

$y° = 180° - w°$

$= 180° - 166°$

$= 14°$

$z° + y° = 63°$

$z° = 63° - y°$

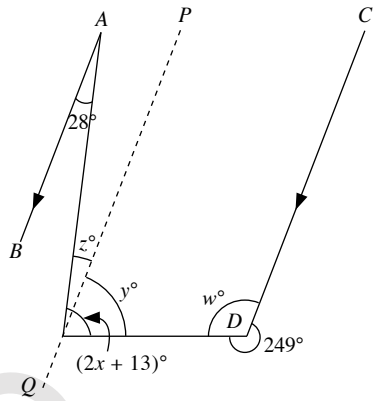
$= 63° - 14°$

f $= 49°$

$x° = z° = 49°$ (alt. ∠s, PQ // DC)

$\therefore x = 49$

(h) Draw a line PQ through E that is parallel to AB and CD.



$w° + 249° = 360°$ (∠s at a point)

$w° = 360° - 249°$

$= 111°$

$y° + w° = 180°$ (int. ∠s, PQ // CD)

$y° = 180° - w°$

$= 180° - 111°$

$= 69°$

$z° = 28°$ (alt. ∠s, AB // PQ)

$(2x + 13)° = y° + z°$

$(2x + 13)° = 69° + 28° = 97°$

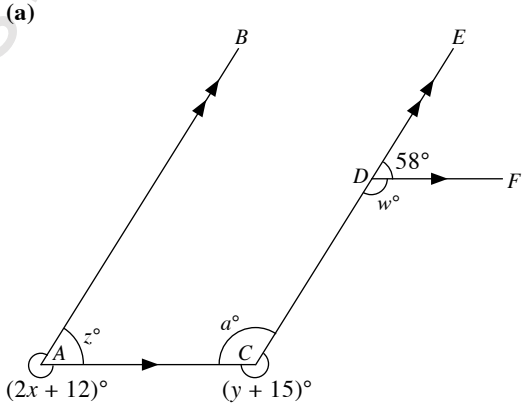
$2x° = 97° - 13°$

$= 84°$

$x° = 42°$

$\therefore x = 42$

8. (a)



$w° + 58° = 180°$ (adj. ∠s on a str. line)

$w° = 180° - 58°$

$= 122°$

$a° = w° = 122°$ (alt. ∠s, DF // AC)

$(y + 15)° + a° = 360°$ (∠s at a point)

$y° + 15° + 122° = 360°$

$y° = 360° - 15° - 122°$

$= 223°$

$z° + a° = 180°$ (int. $\angle$s, AB // CE)

$z° = 180° - a°$

$= 180° - 122°$

$= 58°$

$(2x + 12)° + z° = 360°$ ($\angle$s at a point)

$2x° + 12° + 58° = 360°$

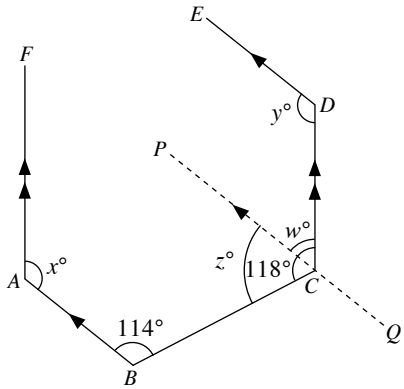
$2x° = 360° - 12° - 58°$

$= 290°$

$x° = 145°$

$\therefore x = 145$ and $y = 223$

(b) Draw a line PQ through C that is parallel to ED and AB.



$z° + 114° = 180°$ (int. $\angle$s, PQ // AB)

$z° = 180° - 114°$

$= 66°$

$w° + z° = 118°$

$w° = 118° - 66°$

$= 52°$

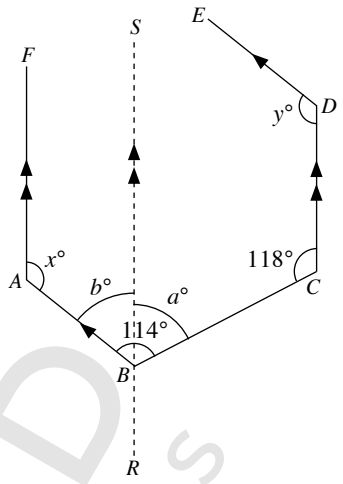
$y° + w° = 180°$ (int. $\angle$s, PQ // ED)

$y° + 52° = 180°$

$y° = 180° - 52°$

$= 128°$

Draw another line SR through B that is parallel to AF and CD.



$a° + 118° = 180°$ (int. $\angle$s, SR // DC)

$a° = 180° - 118°$

$= 62°$

$b° + a° = 114°$

$b° = 114° - a°$

$= 114° - 62°$

$= 52°$

$x° + b° = 180°$ (int. $\angle$s, SR // FA)

$x° = 180° - b°$

$= 180° - 52°$

$= 128°$

$\therefore x = y = 128$

New Trend

9. (i) $W\hat{P}X = 180° - 65° - (180° - 145°)$ (vert. opp. $\angle$s, adj. $\angle$s on a str. line, $\angle$ sum of $\triangle$)
$= 80°$

(ii) Reason 1

Converse of interior angles theorem

Since $W\hat{Y}Z + Y\hat{W}X = 180°$, then $AB \,//\, CD$ (converse of int. $\angle$s)

Reason 2

Converse of corresponding angles postulate

$P\hat{W}X = 180° - 145°$ (adj. $\angle$s on a str. line)
$= 35°$

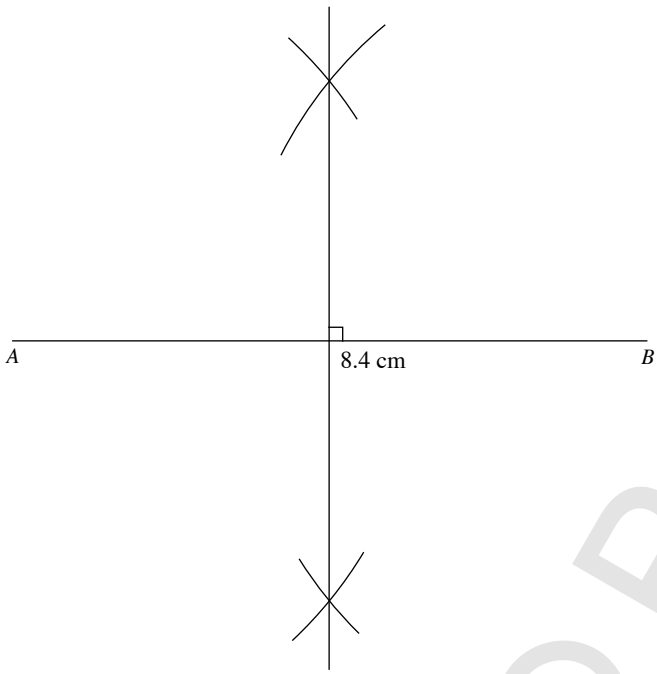
$\therefore$ Since $P\hat{W}X = W\hat{Y}Z$, then $AB \,//\, CD$ (converse of corr. $\angle$s)

(iii) $D\hat{Z}R = B\hat{X}Z$ (corr. $\angle$s, $AB \,//\, CD$)
$= 65°$

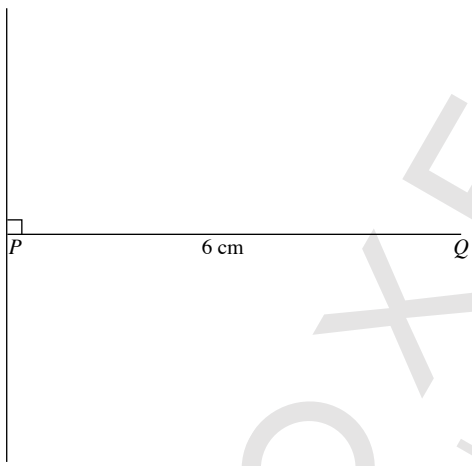
Chapter 11 Geometrical Constructions

Basic

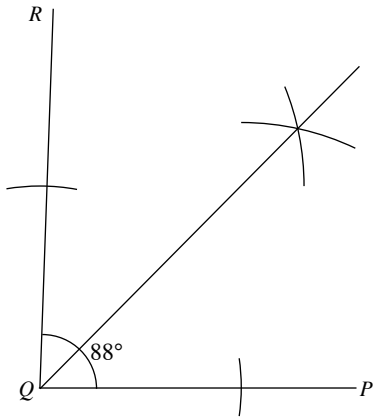
1.



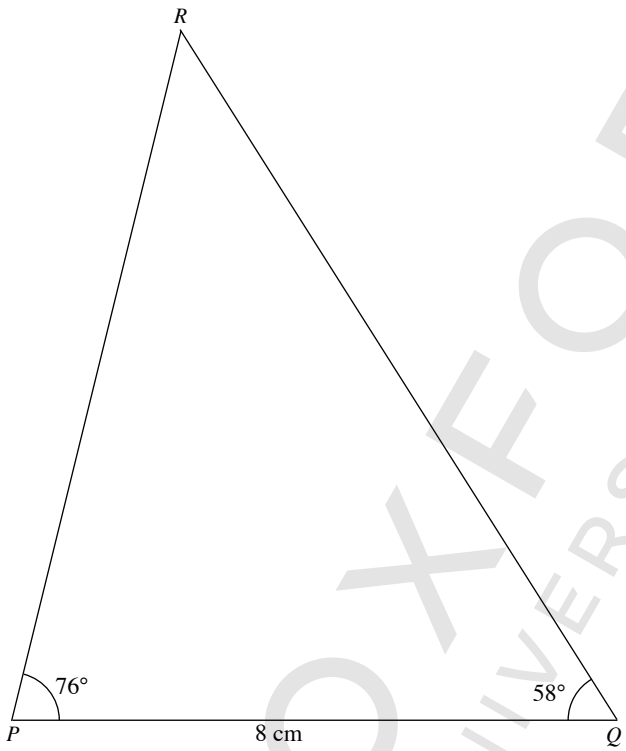
2.



3.



4.

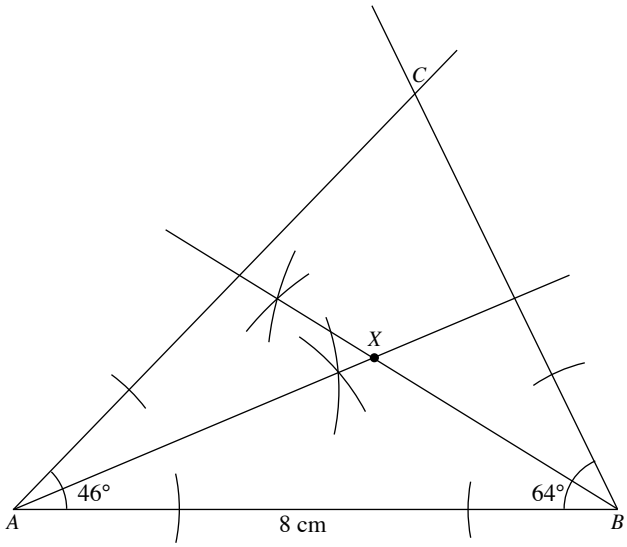


(i) $\angle PRQ = 46°$

(ii) $PR = 9.4$ cm

(iii) $QR = 10.8$ cm

5.



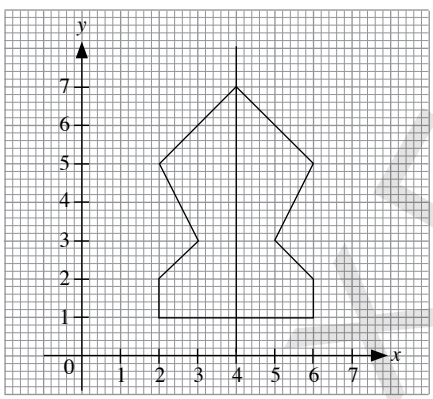
Length of AX = 5.2 cm

Chapter 12 Symmetry

Basic

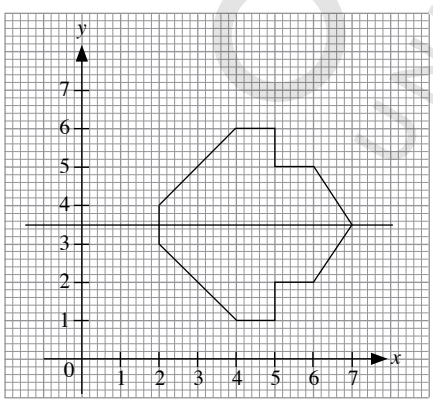
1. **(a)** **(i)** The figure has 1 line of symmetry.
(ii) The figure has rotational symmetry of order 1 i. e. no rotational symmetry.
(b) **(i)** The figure has 1 line of symmetry.
(ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
(c) **(i)** The figure has 2 lines of symmetry.
(ii) The figure has rotational symmetry of order 2.
(d) **(i)** The figure has 0 lines of symmetry, i.e. no line symmetry.
(ii) The figure has rotational symmetry of order 3.
(e) **(i)** The figure has 1 line of symmetry.
(ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
(f) **(i)** The figure has 4 lines of symmetry.
(ii) The figure has rotational symmetry of order 4.
(g) **(i)** The figure has 1 line of symmetry.
(ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.

2. **(a)**



Line of symmetry: $x = 4$

(b)



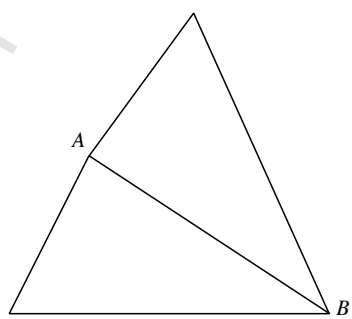
Line of symmetry: $y = 3.5$

3. **(a)** The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
(b) The figure has rotational symmetry of order 5.
(c) The figure has rotational symmetry of order 2.
(d) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
(e) The figure has rotational symmetry of order 4.
(f) The figure has rotational symmetry of order 8.
(g) The figure has rotational symmetry of order 2.

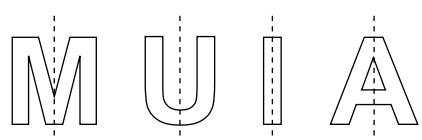
4. **(i)** The letters with line symmetry are O, E, H and I.
(ii) The letters with rotational symmetry are O, S, H and I.

5. **(a)** False
(b) False
(c) True
(d) True
(e) False
(f) True
(g) True
(h) False
(i) True
(j) False
(k) False
(l) False

6. **(a)** An equilateral triangle has 3 lines of symmetry.(b)
(b)



7. **(a)** The letters with a vertical line of symmetry are M, U, I and A.
(b) The letters with horizontal line of symmetry are I and C.
(c) The letter I has two lines of symmetry.
(d) The letters S and L are not symmetrical.
(e)

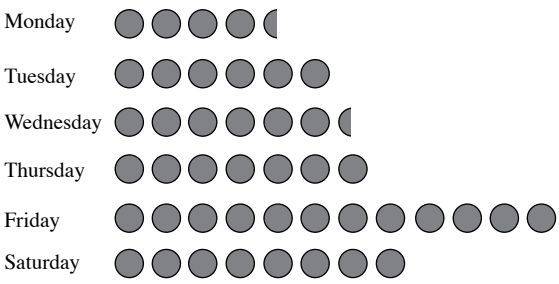


Chapter 13 Statistical Data Handling

Basic

1. (a)

Daily Earnings of ABC Pte Ltd for the Week



Each circle represents PKR 1000.

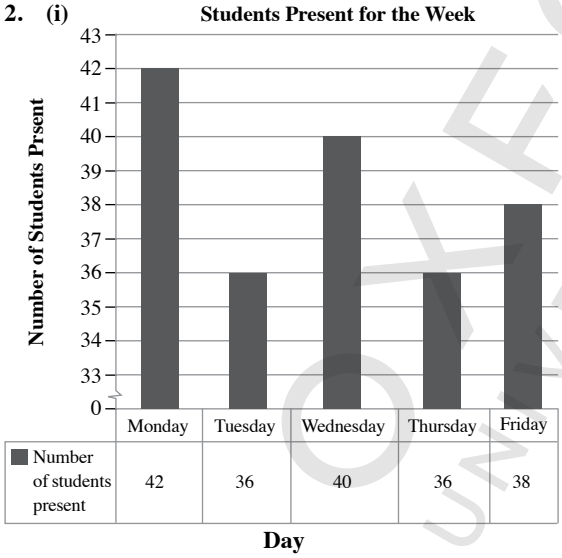
(b) Total earnings for the week
= 4500 + 6000 + 6500 + 7000 + 12 000 + 8000
= PKR 44 000
Percentage of Friday's earning to the total earnings for the week

$$= \frac{12\,000}{44\,000} \times 100\%$$

$$= 27\frac{3}{11}\%$$

2. (i)

Students Present for the Week



(ii) All students were present on Monday.

(iii) Number of absentees on Friday = 42 – 38
= 4
Percentage of absentees on Friday

$$= \frac{4}{42} \times 100\%$$

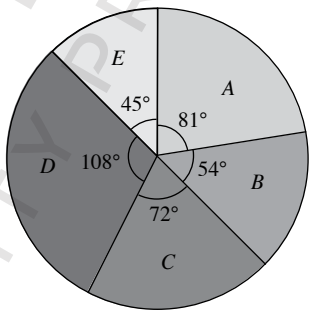
= 9.52% (to 3 s.f.)

(iv) Ahsan is right to say that because on Monday, everyone is present. So, if student *A* is absent from Tuesday to Friday, he is still present at least once in that week and not absent for the whole week.

3. (a) Total number of foreign countries
= 9 + 6 + 8 + 12 + 5 = 40

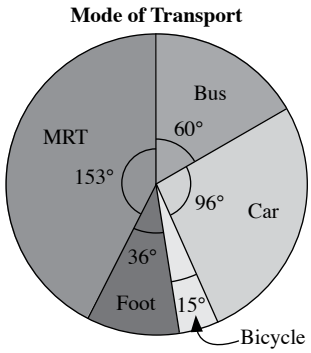
Number of foreign countries	Angle of sector
A	$\frac{9}{40} \times 360° = 81°$
B	$\frac{6}{40} \times 360° = 54°$
C	$\frac{8}{40} \times 360° = 72°$
D	$\frac{12}{40} \times 360° = 108°$
E	$\frac{5}{40} \times 360° = 45°$

Number of Foreign Countries



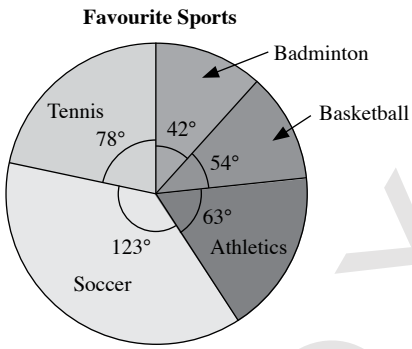
(b) Total number of students surveyed
= 40 + 64 + 10 + 24 + 102 = 240

Mode of Transport	Angle of sector
Bus	$\frac{40}{240} \times 360° = 60°$
Car	$\frac{64}{240} \times 360° = 96°$
Bicycle	$\frac{10}{240} \times 360° = 15°$
Foot	$\frac{24}{240} \times 360° = 36°$
MRT	$\frac{102}{240} \times 360° = 153°$



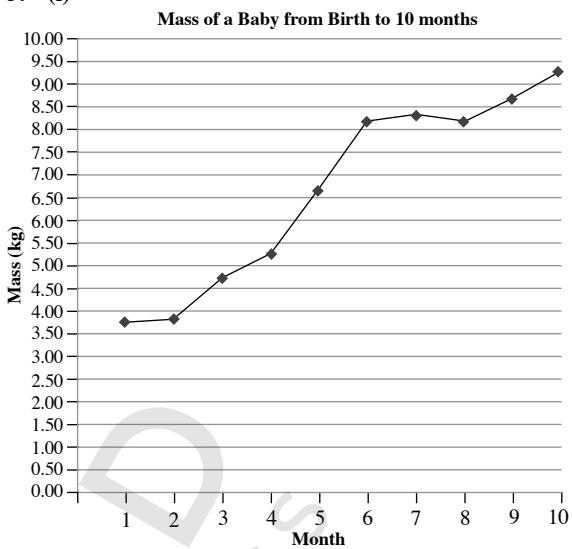
(c)

Sports	Angle of sector
Badminton	$\frac{70}{600} \times 360° = 42°$
Basketball	$\frac{90}{600} \times 360° = 54°$
Athletics	$\frac{105}{600} \times 360° = 63°$
Soccer	$\frac{205}{600} \times 360° = 123°$
Tennis	$\frac{130}{600} \times 360° = 78°$



4. Total number of students in the school
= 30 + 20 + 10 + 20
= 80
Angle of the smallest sector
$= \frac{10}{80} \times 360°$
= 45°
It represents the number of Secondary 3 students in a school for the year 2013.

5. (i)

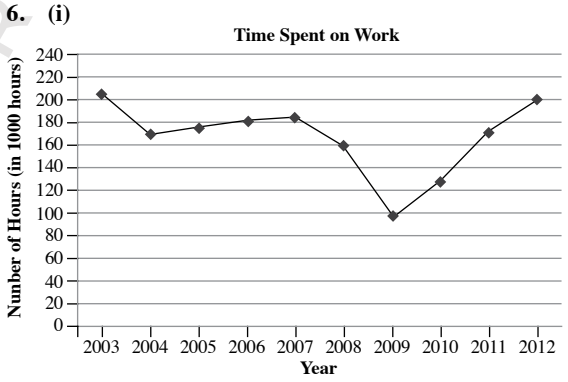


(ii) From the line graph, the increase in the mass of the baby is the largest between the 5th and 6th months.

(iii) From the line graph, the first decrease in the mass is on the 7th month.

(iv) Total mass of the baby from birth to 10 months
= 3.7 + 3.8 + 4.7 + 5.4 + 6.6 + 8.2 + 8.3 + 8.2 + 8.7 + 9.4
= 67 kg
Average mass of the baby
$= \frac{67}{10}$
= 6.7 kg

6. (i)



(ii) The years in which there was a decrease in the number of hours the workers spent in work are 2004, 2008 and 2009.

(iii) The years in which there was an increase in the number of hours the workers spent in work are 2005, 2006, 2007, 2010, 2011 and 2012.

(iv) From the line graph, the year in which the increase is the largest is 2011 and the year in which the increase is the least is 2007.

(v) The possible years in which the workers spent more than 172 000 hours in work are 2003, 2005, 2006, 2007 and 2012.

Intermediate

7. **(i)** **(a)** Number of cars produced on Tuesday
$= 6.5 \times 20$
$= 130$

(b) Number of cars produced on Thursday
$= 5 \times 20$
$= 100$

(c) Number of cars produced on Saturday
$= 0 \times 20 = 0$

(ii) The greatest number of cars produced was on Tuesday.

(iii) Production line has stopped for half a day on Wednesday. One possible indication is that the number of cars produced is low as compared to the other days. The number of cars produced on Wednesday is approximately half the number of cars produced on Monday and on Thursday.

(iv) Increase in production of cars from Monday to Tuesday
$= 130 - 80$
$= 50$
Percentage increase
$= \frac{50}{80} \times 100\%$
$= 62.5\%$

(v) One possible explanation may be the workers are resting on weekends. The other reason may be there may not be orders on weekends and the number of cars produced on Friday may be sufficient to meet the demands for the coming week.

(vi) Total number of cars produced
$= 80 + 130 + 50 + 100 + 120$
$= 480$

8. **(a)** **(i)** Number of students in the class
$= 6 + 7 + 10 + 8 + 4 + 3 + 3$
$= 41$

(ii) Most students are holding 2 coins.

(iii) Total number of coins
$= 7 \times 1 + 10 \times 2 + 8 \times 3 + 4 \times 4 + 3 \times 5 + 3 \times 6$
$= 7 + 20 + 24 + 16 + 15 + 18$
$= 100$
Average number of coins
$= \frac{100}{41}$
$= 2.44$ (to 3 s.f.)

(iv) Number of students having 4 or more coins
$= 4 + 3 + 3$
$= 10$
Percentage of students having 4 or more coins
$= \frac{10}{41} \times 100\%$
$= 24.4\%$ (to 3 s.f.)

(b) Angle representing students having 0 coin
$= \frac{6}{41} \times 360°$
$= 52.7°$
Angle representing students having 1 coin
$= \frac{7}{41} \times 360°$
$= 61.5°$
Angle representing students having 2 coins
$= \frac{10}{41} \times 360°$
$= 87.8°$
Angle representing students having 3 coins
$= \frac{8}{41} \times 360°$
$= 70.2°$
Angle representing students having 4 or more coins
$= \frac{10}{41} \times 360°$
$= 87.8°$

9. **(a)** **(i)** February

(ii) June

(iii) August

(b) The month in which he is the heaviest is in the month of November. His weight is about 54 kg.

(c) The months in which his weights were the same are May, October and December.

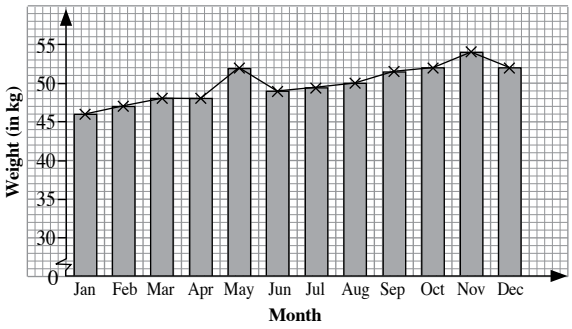
(d) His largest weight = 54 kg
His smallest weight = 46 kg
Range of weight = 54 – 46
= 8 kg

(e) **(i)** On 1st June, he lost weight greatly after his weight increased for the past 5 months. Therefore, he was sick in May.

(ii) On 1st December, he lost weight slightly after his weight increased for the past 5 months. Therefore, he was controlling his diet in November.

(f) October

(g)



(h) Line graph is more suitable to represent and interpret the above data as we can observe the trends of his weight over the months easily.
We can observe the increase or decrease of his weight easily from the line graph.

10. (a) (i) The value of sales in 2007 is 64 × PKR 10 000
= PKR 640 000.

(ii) The value of sales in 2009 is 110 × PKR 10 000
= PKR 1 100 000.

(iii) The value of sales in 2011 is 140 × PKR 10 000
= PKR 1 400 000.

(b) The value of sales is PKR 1 000 000 in 2008.

(c) Between 2009 and 2010, the increase in the value of sales is the greatest.
The maximum value of sales (from 2009 to 2010)
= (160 × PKR 10 000) – PKR 1 100 000
= PKR 500 000

(d) Amount exceeded the sales target
= PKR 1 600 000 – PKR 1 300 000
= PKR 300 000
Percentage of amount exceeded the target
$= \frac{300\,000}{1\,300\,000} \times 100\%$
$= 23\frac{1}{13}\%$

(e) Amount below the sales target
= PKR 1 650 000 – PKR 1 400 000
= PKR 250 000
Percentage of amount below the target
$= \frac{250\,000}{1\,650\,000} \times 100\%$
$= 15\frac{5}{33}\%$

(f) Total value of sales over the past 6 years
= (64 + 100 + 110 + 160 + 140 + 50) × PKR 10 000
= PKR 6 240 000

(g) The sudden increase may be due to the increase in the popularity of the product. Another reason may be the population in the country has increased over the past year and the demand for the product increases as it is a necessity.

11. (i) $5x° + 2x° + 52° = 360°$
$7x° + 52° = 360°$
$7x° = 308°$
$x° = 44°$
$\therefore x = 44$

(ii) $2 \times 44° = 88°$ represents 66 vehicles
1° represents 0.75 vehicles
360° represents 0.75 × 360 = 270 vehicles
The total number of vehicles included in the survey is 270.

12. (i) When it rained the whole day, the average temperature should be the lowest among the 10 days. In this case, the day in which it rained the whole day is Monday during the 1st week and its temperature is 24°C.

(ii) Friday, the 1st week; the temperature in the classroom on that day is 31°C.

(iii) The days when the temperature is below 29°C are 1st week on Monday, Tuesday and Wednesday and 2nd week on Friday.

(iv) Number of days in which the temperature is above 28°C
= 6
Percentage of days in which the temperature is above 28°C
$= \frac{6}{10} \times 100\%$
= 60%

(v) The sudden increase in temperature may be due to a change in weather. Another reason may be the monsoon season has ended and the temperature has resumed to its initial temperature before the monsoon season.

13. (i) The sale first exceeds the 50 000 mark in year 2010.

(ii) In year 2012, the sale was exactly 100 000.

(iii) Between 2011 and 2012, the sales in the soap powder were the greatest.

(iv)

Year	**2008**	**2009**	**2010**	**2011**	**2012**
Number of Packets (in thousands)	40	40	60	70	100

(v) Increase in sales from 2010 to 2012

= 100 000 – 60 000

= 40 000

Percentage increase in sales from 2010 to 2012

$= \frac{40\,000}{60\,000} \times 100\%$

$= 66\frac{2}{3}\,\%$

Advanced

14. Yes.

Suggested answer:

The increase in the size of the diagram does not represent accurately that the sales have increased by 300%. What the advertisement is trying to show is that there is an increase in the sales but it is unable to represent the increase as 300%.

15. No.

Suggested answer:

The charts did show an increase in the radius of the circle by two times. However, the actual figures of the sales are not given. Therefore, it is not conclusive that the sales have doubled from the year 2010 to 2012.

Suggestion: A better representation is a bar graph which compares the sales in 2010 and 2012 using bars.

16. No. I do not agree with Amirah. The person who collected the data did not mention whether taking more projects of the same nature contributes to people involved in more community work.

Reason 1: More people may have increased their involvement from May to June by taking part in more projects within the same organisation. Therefore, the nature of the projects may not have changed but the number of projects involved has increased.

Reason 2: There may be a higher chance of people involving in community work due to demand for more volunteers as part of the school's holiday programmes.

New Trend

17. (a) Number of females who use public transport

= 55 – 20 – 2 – 9

= 24

(b) Angle representing students walking to school

$= \frac{16}{120} \times 360°$

= 48°

(c) For males,

percentage who travel using other modes of transport

$= \frac{12}{65} \times 100\%$

= 18.462% (to 5 s.f.)

For females,

percentage who travel using other modes of transport

$= \frac{9}{55} \times 100\%$

= 16.364% (to 5 s.f.)

Difference in percentage = 18.462% – 16.364%

= 2.10%

A greater percentage travel using other modes of transport in males as compared to females. The percentage in males is 2.10% higher.

18. (a) Ratio of manufactured goods and the minerals

$= \frac{85}{115}$

$= \frac{17}{23}$

= 17 : 23

(b) Angle representing agricultural produce

= 360° – 10° – 85° – 115°

= 150°

Ratio of agricultural produce and the manufactured goods

$= \frac{150}{85}$

$= \frac{30}{17}$

= 30 : 17

(c) 115° represent 23 million

1° represents 0.2 million

360° represent 72 million

The total value of exports of the country is 2012 is 72 million.

Chapter 14 Averages of Statistical Data

Basic

1. **(a)** 11, 11, 12, 13, 16

$$\text{Mean} = \frac{11 + 11 + 12 + 13 + 16}{5}$$
$$= 12.6$$

Median = 12

Mode = 11

(b) 11, 12, 18, 18, 20, 20, 20, 24, 29, 41

$$\text{Mean} = \frac{11 + 12 + 18 + 18 + 20 + 20 + 20 + 24 + 29 + 41}{10}$$
$$= 21.3$$

$$\text{Median} = \frac{20 + 20}{2}$$
$$= 20$$

Mode = 20

(c) 10.5, 12.6, 12.6, 13.5, 14.3, 15.3, 16.0, 16.4

$$\text{Mean} = \frac{10.5 + 12.6 + 12.6 + 13.5 + 14.3 + 15.3 + 16.0 + 16.4}{8}$$
$$= 13.9$$

$$\text{Median} = \frac{13.5 + 14.3}{2}$$
$$= 13.9$$

Mode = 12.6

(d) 7, 8.1, 8.1, 8.1, 9.4, 9.4, 9.6, 10.4, 10.5, 11, 11.7

$$\text{Mean} = \frac{7 + 8.1 + 8.1 + 8.1 + 9.4 + 9.4 + 9.6 + 10.4 + 10.5 + 11 + 11.7}{11}$$
$$= 9.39 \text{ (to 3 s.f.)}$$

Median = 9.4

Mode = 8.1

2. 35, 36, 38, 38, 38, 39, 39, 40, 42, 43, 45, 45, 45, 45, 47

(i) $$\text{Mean} = \frac{35 + 36 + 38 + 38 + 38 + 39 + 39 + 40 + 42 + 43 + 45 + 45 + 45 + 45 + 47}{15}$$
$$= 41$$

(ii) Mode = 45

(iii) Median = 40

3. $$\text{Mean} = \frac{3 + 7 + 13 + 14 + 16 + 19 + 20 + x}{8}$$
$$= \frac{92 + x}{8}$$

$$\text{Median} = \frac{14 + 16}{2}$$
$$= 15$$

Since mean = median,

$$\frac{92 + x}{8} = 15$$
$$92 + x = 120$$
$$x = 28$$

4. **(i)** Total number of seeds = 100 × 5
= 500

(ii) Number of seeds that germinated
= 30 × 1 + 25 × 2 + 20 × 3 + 10 × 4 + 5 × 5
= 205

$$\text{Fraction of seeds that germinated} = \frac{205}{500}$$
$$= \frac{41}{100}$$

(iii) $$\text{Mean} = \frac{10 \times 0 + 30 \times 1 + 25 \times 2 + 20 \times 3 + 10 \times 4 + 5 \times 5}{100}$$
$$= 2.05$$

Median = 2

Mode = 1

Intermediate

5. Let the eighth number be x.

1, 2, 2, 4, x, 7, 8, 13

$$\text{Median} = \frac{4 + x}{2}$$
$$4.5 = \frac{4 + x}{2}$$
$$9 = 4 + x$$
$$x = 5$$

$\therefore$ The eighth number is 5.

Mode = 2

6. Sum of the set of 12 nunbers = 12 × 5
= 60

Sum of the set of 8 numbers = $8a$

$$\text{Mean of combined set of 20 numbers} = \frac{60 + 8a}{20}$$
$$8 = \frac{60 + 8a}{20}$$
$$160 = 60 + 8a$$
$$8a = 100$$
$$a = \frac{100}{8}$$
$$= 12.5$$

7. **(a)** **(i)** Modal profit = PKR 3 million

(ii) Median profit = PKR 2 million

(b) Mean profit

$$= \frac{2 \times 0 + 6 \times 1 + 8 \times 2 + 10 \times 3 + 4 \times 4}{30}$$

= PKR 2.27 million (to 3 s.f.)

∴ Ali is incorrect.

8. Initial sum of eye pressure = 30 × 12.4

= 372 mm Hg

New sum of eye pressure = 30 × 12.6

= 378 mm Hg

∴ Nadia's actual eye presssure = 8 + (378 – 372)

= 14 mm Hg

9. 62.0, 62.0, 62.6, 63.1, 63.7, 64.2, 64.3, 64.7, 65.1, 65.2, 65.2, 65.2, 65.5, 65.9, 66.8, 67.1, 67.4, 68.2

(a) **(i)** Mean $= \frac{\begin{array}{l}62.0 + 62.0 + 62.6 + 63.1 + 63.7 \\ + 64.2 + 64.3 + 64.7 + 65.1 + 65.2 \\ + 65.2 + 65.2 + 65.5 + 65.9 + 66.8 \\ + 67.1 + 67.4 + 68.2\end{array}}{18}$

= 64.9 s

(ii) Mode = 65.2 s

(iii) Median $= \frac{65.1 + 65.2}{2}$

= 65.15 s

(b) Percentage $= \frac{62.0}{68.2} \times 100\%$

= 90.9 %

10. 1.6, 1.7, 1.8, 1.8, 1.8, 1.8, 1.9, 1.9, 1.9, 2.0, 2.0

(a) **(i)** Modal height = 1.8 m

(ii) Median height = 1.8 m

(iii) Mean height $= \frac{\begin{array}{l}1.6 + 1.7 + 1.8 + 1.8 + 1.8 + 1.8 \\ + 1.9 + 1.9 + 1.9 + 2.0 + 2.0\end{array}}{11}$

$= \frac{20.2}{11}$

= 1.84 m (to 3 s.f.)

(b) Sum of heights of the first 11 boys = 20.2 m

Sum of heights of the 12 boys = 12 × 1.85

= 22.2 m

∴ Height of the 12th boy = 22.2 – 20.2

= 2.0 m

11. **(i)** Total number of pages = 1 + 3 + 10 + 7 + 4 + 3 + 2

= 30

(ii) Number of pages with fewer than 3 errors

= 1 + 3 + 10

= 14

Percentage of pages with fewer than 3 errors

$= \frac{14}{30} \times 100\%$

= 46.7% (to 3 s.f.)

(iii) Mode = 2

(iv) Mean $= \frac{\begin{array}{l}1 \times 0 + 3 \times 1 + 10 \times 2 + 7 \times 3 \\ + 4 \times 4 + 3 \times 5 + 2 \times 6\end{array}}{30}$

= 2.9

12. **(i)** Total number of days = 3 + 5 + 8 + 7 + 10 + 6 + 1

= 40

(ii) Mean number of security cameras sold

$= \frac{\begin{array}{l}3 \times 32 + 5 \times 57 + 8 \times 82 + 7 \times 107 \\ + 10 \times 132 + 6 \times 157 + 1 \times 182\end{array}}{40}$

= 105.75

(iii) Median = 107

Mode = 132

∴ The median gives a better comparison.

Advanced

13. Total mass of the children

= 15 + 15 + 11 + 13 + 9 + 20 + 15 + a + 13 + 18

= 129 + a

Mean mass of the children $= \frac{129 + a}{10}$

Arrangement of the masses without a:

9, 11, 13, 13, 15, 15, 15, 18, 20

X X X X Y Z Z Z Z Z

Case 1: a lies at one of the points labelled X.

Median = 14

$\frac{129 + a}{10} = 14 - 0.4$

$129 + a = 136$

$a = 7$

Case 2: a lies at the point labelled Y.

Median $= \frac{a + 15}{2}$

$\frac{129 + a}{10} = \frac{a + 15}{2} - 0.4$

$129 + a = 5a + 75 - 4$

$4a = 58$

$a = 14.5$

Case 3: a lies at one of the points labelled Z.

Median = 15

$\frac{129 + a}{10} = 15 - 0.4$

$129 + a = 146$

$a = 17$

∴ $a = 7$ or $a = 14.5$ or $a = 17$

New Trend

14. Let the numbers be x, y, 60 and 60, such that $x < y$.

Since the median is 56,

$$\frac{y + 60}{2} = 56$$
$$y + 60 = 112$$
$$y = 52$$

Since the mean is 54,

$$\frac{x + 52 + 60 + 60}{4} = 54$$
$$x + 172 = 216$$
$$x = 44$$

$\therefore$ The four numbers are 44, 52, 60 and 60.

15. (a) Difference = 100 – (–210)
= 310°C

(b) Mean boiling points $= \frac{2856 + 100 + (-195.79)}{3}$
$= \frac{2760.21}{3}$
= 920.07°C (to 2 d.p.)

Mean melting points $= \frac{1064.18 + 0 + (-210)}{3}$
$= \frac{854.18}{3}$
= 284.73°C (to 2 d.p.)

Chapter 15 Probability of Single Events

Basic

1. **(a)** $\{A_1, A_2, C, E, H, I, M_1, M_2, S, T_1, T_2\}$

(b) **(i)** Probability of obtaining the letter 'A' $= \frac{2}{11}$

(ii) Probability of obtaining the letter 'H' $= \frac{1}{11}$

(iii) Probability of obtaining a vowel $= \frac{4}{11}$

2. **(i)** Probability of getting an odd number $= \frac{3}{6}$

$= \frac{1}{2}$

(ii) Probability of getting a number less than 4 $= \frac{3}{6}$

$= \frac{1}{2}$

3. **(i)** Probability of drawing a number that is a multiple of 3 $= \frac{5}{8}$

(ii) Probability of drawing a prime number $= \frac{2}{8}$

$= \frac{1}{4}$

(iii) Probability of drawing a number whose digits have a sum that is divisible by 2 $= \frac{3}{8}$

4. **(i)** Number of white pearls $= 50 - 24 - 15 = 11$

Probability of selecting a white pearl $= \frac{11}{50}$

(ii) Probability that the pearl selected is not green

$= \frac{24 + 11}{50}$

$= \frac{35}{50}$

$= \frac{7}{10}$

(iii) Probability of selecting a pink pearl $= 0$

5. **(i)** Probability that the month is December $= \frac{1}{12}$

(ii) Probability that the month begins with the letter J

$= \frac{3}{12}$

$= \frac{1}{4}$

(iii) Probability that the month has exactly 30 days

$= \frac{4}{12}$

$= \frac{1}{3}$

6. **(i)** Probability that a bag selected has a mass of exactly 1 kg $= 1 - \frac{1}{40} - \frac{1}{160}$

$= \frac{31}{32}$

(ii) Number of bags each with a mass of less than 1 kg

$= \frac{1}{160} \times 8000$

$= 50$

Intermediate

7. **(i)** Number of slots $= 37$

Probability that the ball lands in the slot numbered 13 $= \frac{1}{37}$

(ii) Prime numbers from 0 to 37: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

Probability that the ball lands in the slot numbered with a prime number $= \frac{11}{37}$

(iii) Probability that the ball lands in the slot numbered with a number less than 19 $= \frac{19}{37}$

(iv) Probability that the ball lands in the slot numbered with an odd number $= \frac{18}{37}$